Penetration of elastic sheets is an ubiquitous mode of failure which is related to conventional cavitation but differs from it in that the latter is induced by boundary data whereas the former is induced by contact. In particular, materials that do not support conventional cavitation may nevertheless be perforated in this manner. In this work an energy argument similar to that used in Fracture Mechanics is used to predict the penetration of an elastic membrane by a rigid cylinder. A sequence of equilibrium states is studied for a particular material model, beginning with indentation, followed by penetration, and ending in a state of conventional cavitation under suitable boundary data. The analysis is used to construct a failure envelope relating boundary displacement at the edge to the displacement of the indenter at incipient penetration.