Budget Analysis for Reactive Plume Transport over Urban Roughness

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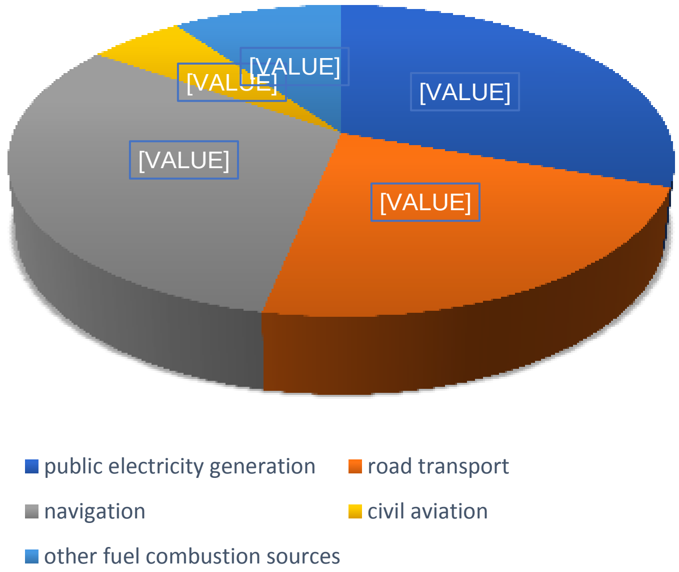
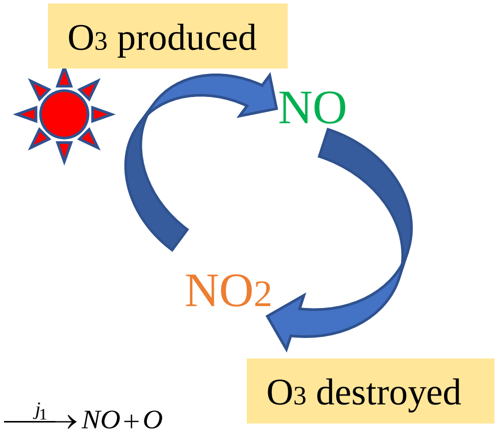
Background

- Buildings and population largely affect the air quality and even the local urban climate in many metropolises.
- Accurate modelling of the interface between an urban surface and the atmosphere will benefit applications such as weather forecasting, air quality and sustainable urban development.
- Hong Kong is a city with high density of population and buildings.

Background





$$NO_2 + h\nu \xrightarrow{j_1} NO + O$$

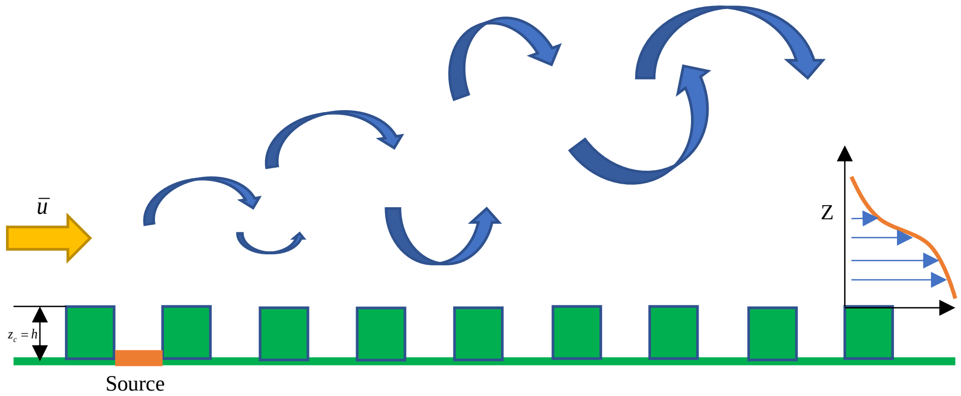
$$O + O_2 + M \xrightarrow{k_2} O_3 + M$$

$$O_3 + NO \xrightarrow{k_3} NO_2 + O_2$$

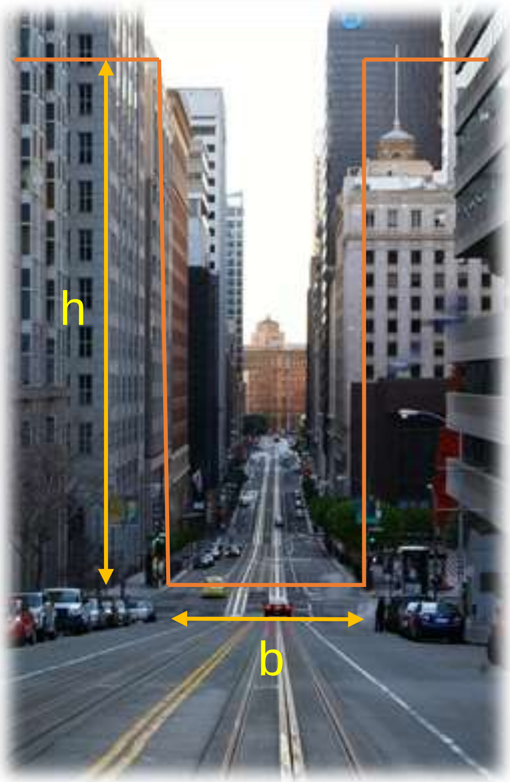
- According to Hong Kong (HK) Environmental Protection Department (EPD), the total emission of NOx in HK in 2013 was about 113,220 tonnes.
- The emission sources of NO_x in HK include public electricity generation, road transport, navigation, civil aviation and other fuel combustion sources, etc.
- Road transport is one of the major sources of NO_x.

- While most practical dispersion models assume inert pollutants, emissions from traffic exhaust are chemically reactive.
- NO can be oxidized by ozone (O_3) in the atmosphere.
- Under sunlight, NO_2 can also be decomposed into NO and O_2
- Chemical reactions in the atmosphere are much more complicated.

Background



- Street canyon is a basic unit commonly used in CFD model to investigate the mechanism of flows over roughness.
- Dynamics are complicated by atmospheric turbulence, geometry/orientation of buildings, thermal stratification and chemical kinetics, etc.
- Oxidation rate of NO is affected by both physical and chemical processes. The physical process is mixing of the plume with the ambient air. The chemical processes are the molecular reactions of NO with different species in the atmosphere.
- There exists a wide range of turbulent eddies which act on plume dispersion. Dispersion and mixing of materials, which in fact constitute the plume, are driven by eddies of all sizes in the atmospheric boundary layer.
  - Large eddies : cause plume meandering.
  - Middle eddies : cause widening and internal mixing of plume on small scales.
  - Small eddies: important for chemical reactions.



Street canyon
Building-height-to-street-width (aspect) ratio
AR = h/b

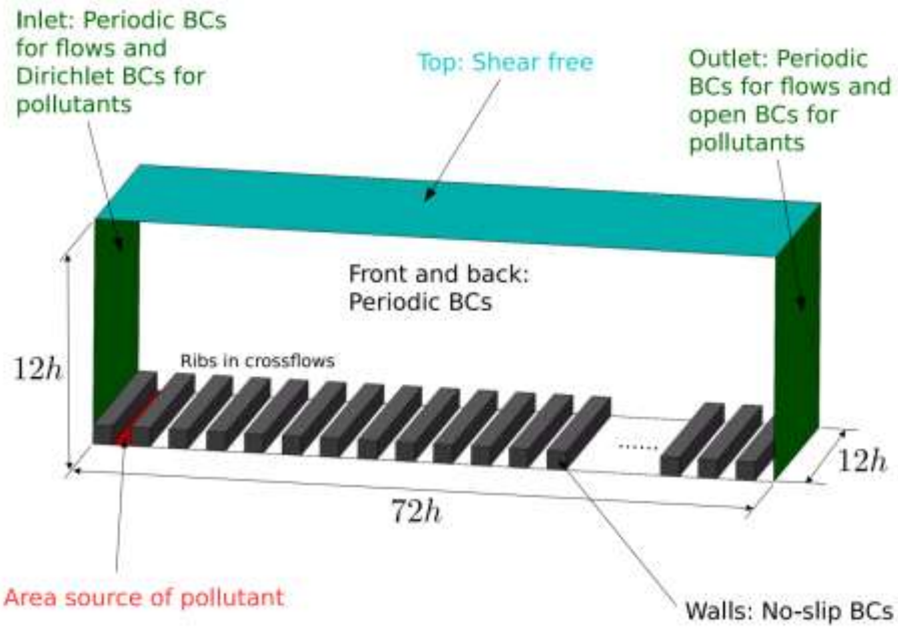
Objective

- Develop a CFD model for simple NO_x-O_3 chemistry.
- Contrast the plume dispersion characteristics of passive scalar and chemically reactive pollutant.
- Analyze the plume characteristics in scenarios with different O_3 background concentrations using Gaussian plume model.
- Elucidate the contribution of advection, diffusion and chemistry term in pollutant transport.

Methodology

- Model
  Large-eddy simulation (LES) with one-equation SGS model
- Governing equations (filtered)
  - continuity
    $$\frac{\partial \bar{u}_i}{\partial x_i} = 0$$
  - momentum conservation
    $$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j}\bar{u}_i \bar{u}_j = -\Delta P \delta_{i1} - \frac{\partial \bar{p}}{\partial x_i} + (\nu + \nu_{SGS})\frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j}$$
  - Transport equation of pollutants
    $$\frac{\partial \bar{\phi}}{\partial t} + \bar{u}_j \frac{\partial \bar{\phi}}{\partial x_j} = \frac{\partial}{\partial x_j}\left(D\frac{\partial \bar{\phi}}{\partial x_j}\right) + S(\bar{\phi})$$
- First step is to handle the irreversible chemical reaction
  $$NO + O_3 \xrightarrow{k_3} NO_2 + O_2$$
- Source terms for NO, NO_2 and O_3
  $$\frac{d[NO]}{dt} = -k_3[O_3][NO] \qquad \frac{d[O_3]}{dt} = -k_3[O_3][NO] \qquad \frac{d[NO_2]}{dt} = k_3[O_3][NO]$$
- The open-source CFD tool OpenFOAM is used to simulate. Finite volume method (FVM) is used to solve the governing equations. The implicit second-order-accurate backward differencing is using in the temporal domain.

CFD Model



Case NO.	NO Concentration /ppb	O_3 Concentration /ppb
Case 1	1000	1
Case 2	1000	10
Case 3	1000	50
Case 4	1000	100
Case 5	1000	200
Case 6	1000	500

- The LES model for hypothetical urban area consists of a number of idealized urban street canyons fabricated by identical square bars of size h .
- The spatial domain sizes 72h (length) × 12h (width) × 12h (height) that is composed of 36 idealized street canyons of the same geometry.
- The street width b is the same as the building height h so the building-height-to-street-width (aspect) ratio is equal to unity that falls into the skimming flow regime (Oke 1988).
- The prevailing flows in the urban ABL are driven by the (background) pressure gradient ΔP_x perpendicular to the street axes, representing the worst scenario of pollutant removal from street canyons.

Results: Flow Validated by Wind Tunnel Experiment

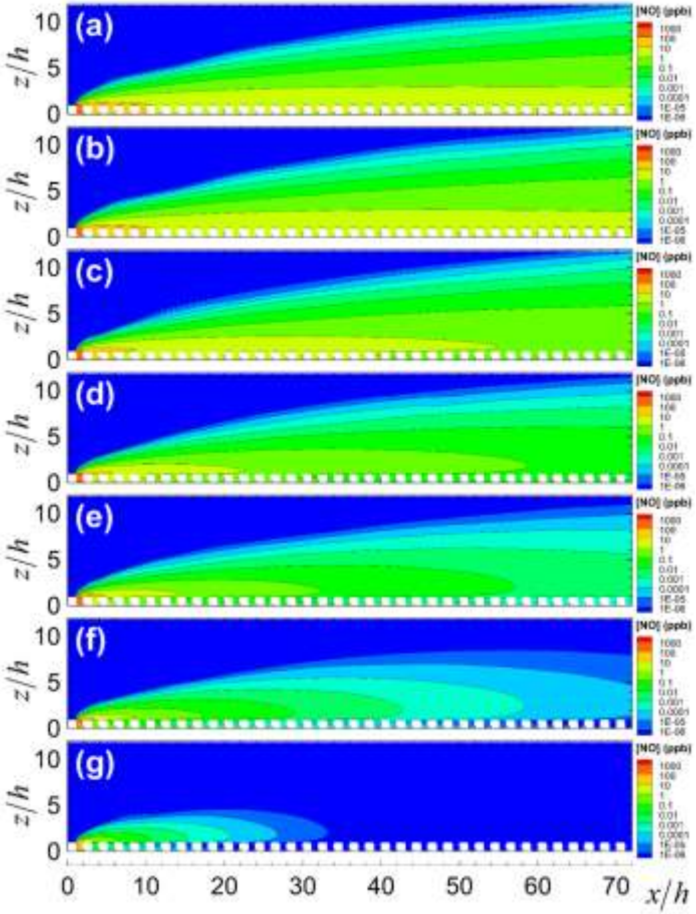


Figure 1. Concentration contours of (a) passive scalar (unit: ppb); (b) NO in case $[O_3]_0$ = 1 ppb; (c) NO in case $[O_3]_0$ = 10 ppb; (d) NO in case $[O_3]_0$ = 50 ppb; (e) NO in case $[O_3]_0$ = 100 ppb; (f) NO in case $[O_3]_0$ = 200 ppb; and (g) NO in case $[O_3]_0$ = 500 ppb.

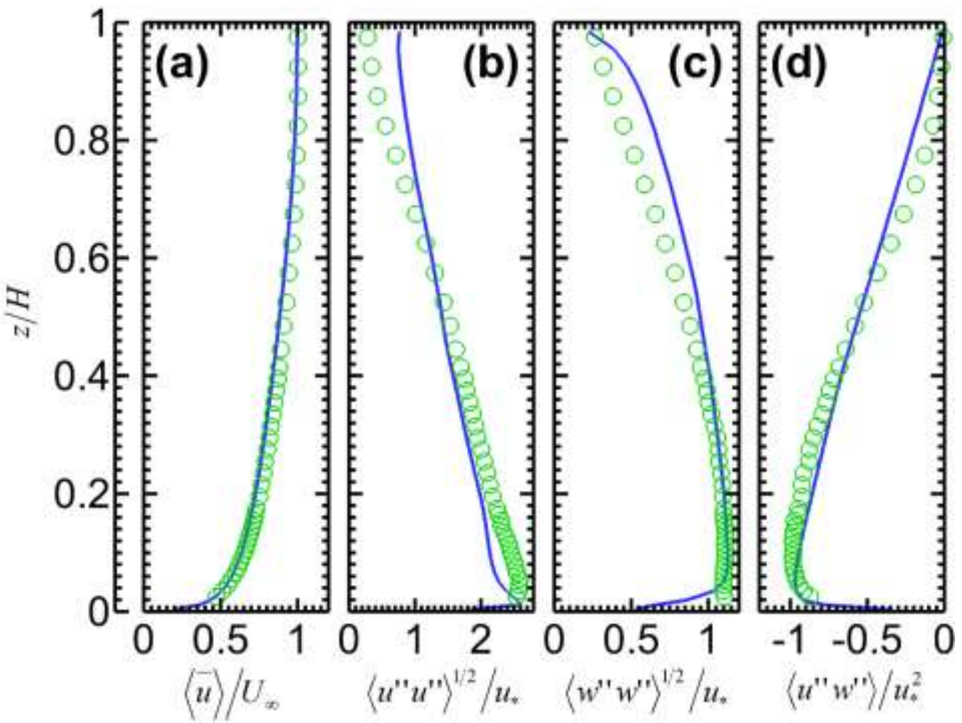
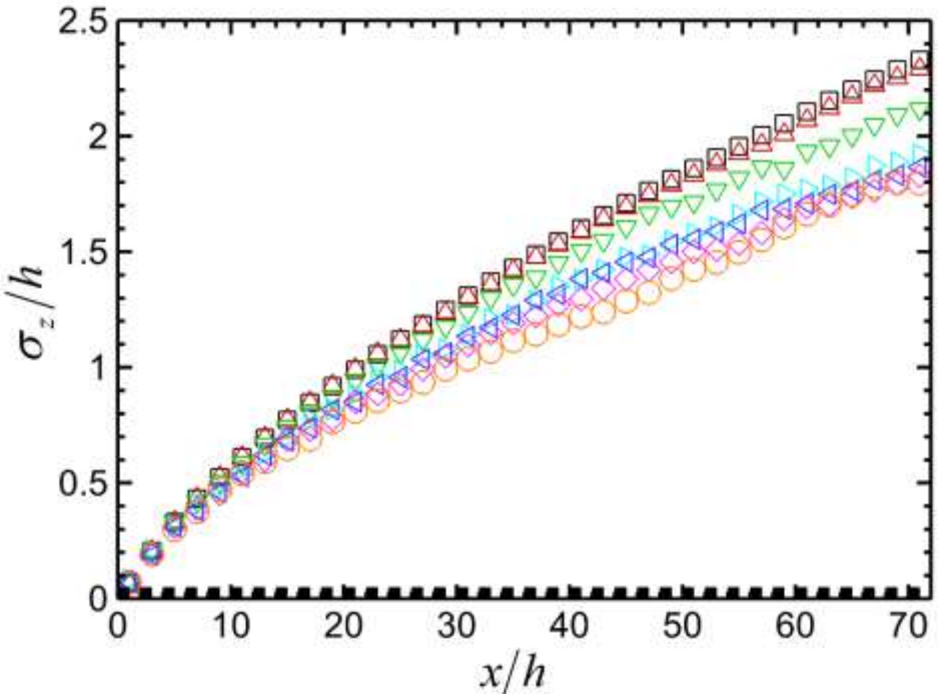
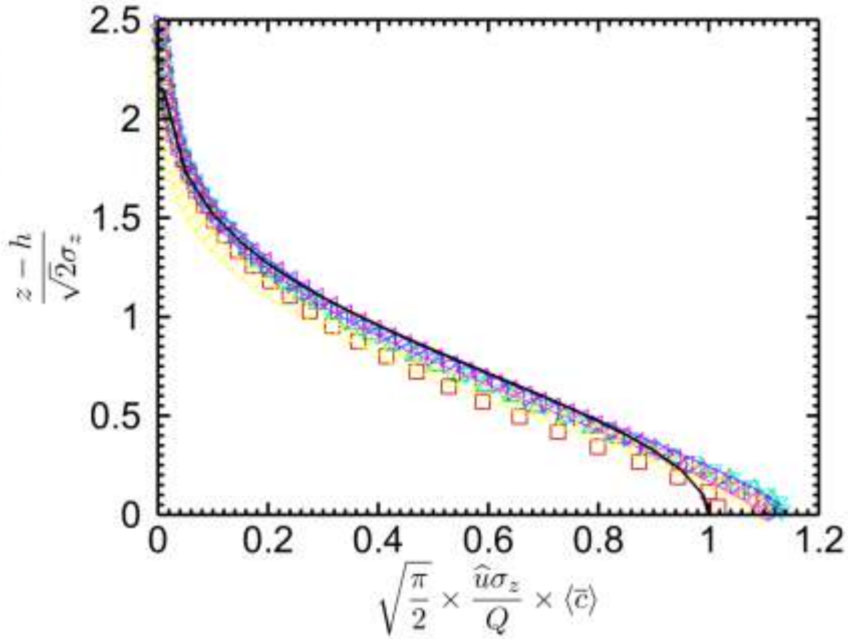


Figure 2. Vertical profiles of (a) mean wind speed; (b) streamwise fluctuating velocity; (c) vertical fluctuating velocity and (d) vertical momentum flux. Solid line: LES data; symbols: wind tunnel data (Ho and Liu 2016).

Results: Plume Characteristic



Dispersion coefficient σ_z of passive scalar (□) and NO in case $[O_3]_0$ = : 1 ppb (△), 10 ppb(▽), 50 ppb(▻), 100 ppb(◅), 200 ppb(◇), 500 ppb(○).



Profiles of dimensionless concentration of passive scalar at x/h =: (a) 15.5 (□), (b) 25.5 (△), (c) 35.5 (▽), (d) 45.5 (▻), (e) 55.5 (◅) and (f) 65.5 ◇). Theoretical Gaussian plume profile (——)

- To parameterize the plume characteristic, the dispersion coefficient σ_z is adopted to represent the length scale of the plume coverage.
- Dimensionless profiles of passive scalar concentration is depicted as functions of height z at different streamwise locations x. The current-LES calculated passive scalar concentration agrees well with the theoretical Gaussian-form solution.

$$\sigma_z = \sqrt{\frac{\iiint (z - z_c)^2 \langle \bar{\phi} \rangle \, dydtdz}{\iiint \langle \bar{\phi} \rangle \, dydtdz}}$$

$$C(x,z) = \frac{Q}{(\pi/2)^{1/2} U \sigma_z(x)} \exp\left[-\frac{1}{2} \left(\frac{z-h}{\sigma_z(x)} \right)^2 \right]$$

Results: Plume Characteristic

- For passive scalar, Gaussian form fits well. For the chemically reactive pollutant, Gaussian form cannot explain the region below the plume center. Above the plume center, Gaussian form can still fit well.

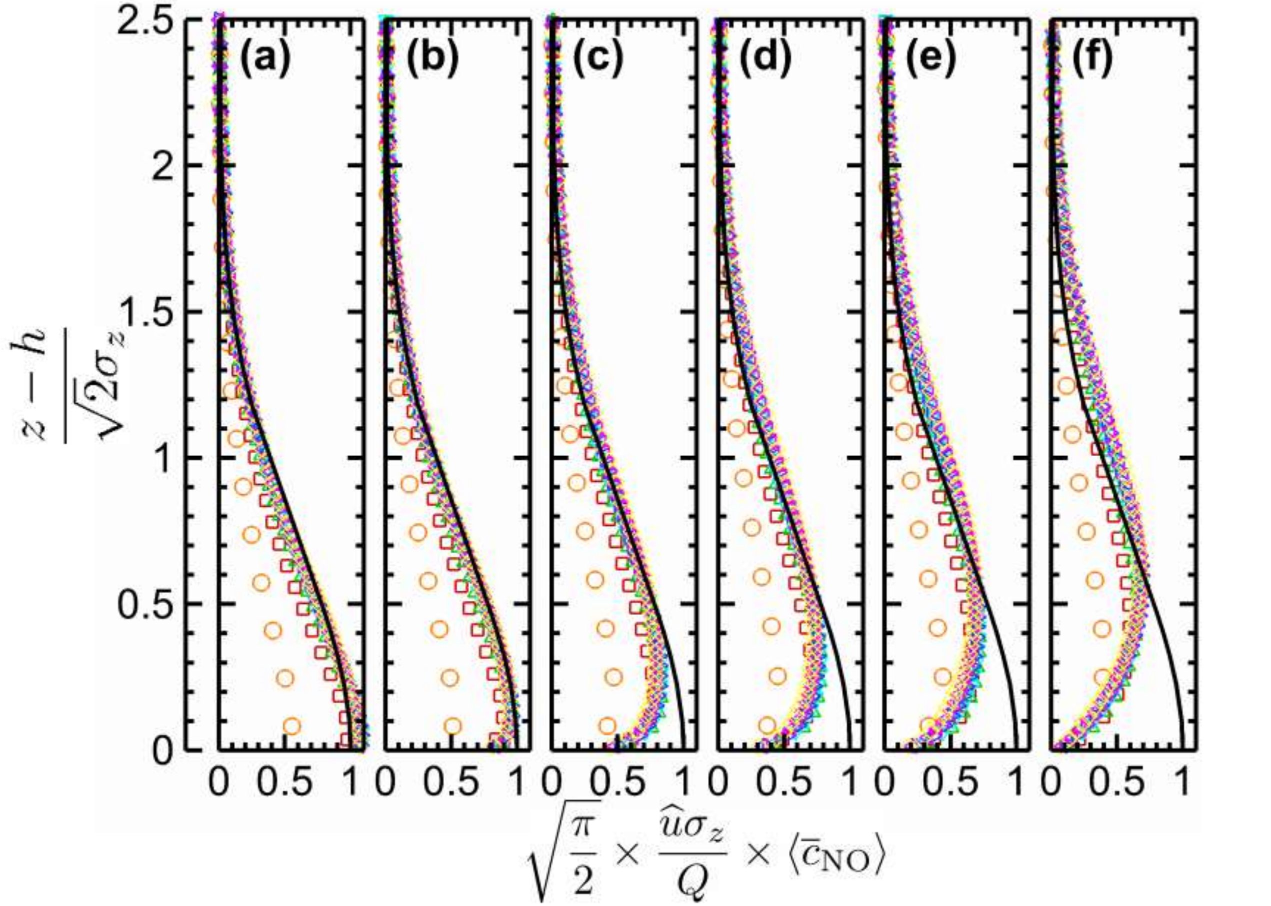


Figure 5: Profiles of dimensionless concentration of reactive pollutant NO at x/h =: 15.5 (□), 25.5 (△), 35.5 (▽), 45.5 (▷), 55.5 (◁) and 65.5 (○) for background ozone concentration $[O_3]_0$ = (a) 1 ppb, (b) 10 ppb, (c) 50 ppb, (d) 100 ppb, (e) 200 ppb and (f) 500 ppb. Also shown is the theoretical Gaussian plume profile (——).

NO Transport in Reactive Plume

- The Convection-Diffusion equation with source/sink term:

$$\frac{\partial \phi}{\partial t} + U_i \frac{\partial \phi}{\partial x_i} = K \frac{\partial^2 \phi}{\partial x_i^2} + S_\phi$$

Advection Term | Diffusion Term | Source/Sink Term

- The simple ozone titration is used as the chemistry model:

$$\mathrm{NO} + \mathrm{O}_3 \xrightarrow{k_3} \mathrm{NO}_2 + \mathrm{O}_2$$

$$\frac{d[NO]}{dt} = -k_3[O_3][NO]$$

$$\frac{d[O_3]}{dt} = -k_3[O_3][NO]$$

Sink of NO and O3

$$\frac{d[NO_2]}{dt} = k_3[O_3][NO]$$

Source of NO2

NO Transport in Reactive Plume

- Take NO as an example, Neglect molecular diffusion for simple dilution, which should be reasonable in turbulent flow.

$$\frac{\partial [NO]}{\partial t} + U_i \frac{\partial [NO]}{\partial x_i} = -k_3 [NO][O_3]$$

- Define characteristic scale

Symbol	$[NO]_0$	$[O_3]_0$	L_0	U_0	T_0
Description	Characteristic concentration scale for NO	Characteristic concentration scale for O_3	Characteristic length scale	Characteristic velocity scale	Characteristic time scale for transport

- Time scales

$$T_0 = L_0 / U_0$$

Advection

$$\tau_{NO}^{-1} = \frac{\partial [NO]}{\partial t} \times \frac{1}{[NO]} = \frac{\partial [NO]}{\partial t} \times \frac{1}{[NO]_0} = -k_3 [O_3]_0$$

Chemistry timescale

NO Transport in Reactive Plume - Damköhler number

- The Damköhler number Da is to compare the time scales of the physical diffusion and chemical reaction.

$$Da_{NO} = T_0 / \tau_{NO} = L_0 k_3 [O_3] / U_0$$

  - Da>1: The chemical reaction is faster than the physical diffusion. Although the chemical reactions are fast, the pollutants are not mixed uniformly for complete reactions.
  - Da<1: the physical diffusion is faster than its chemical counterparts, slow chemical reactions occur in the well-mixed condition.

- Take the ensemble-average :

$$\left\langle \frac{\partial \hat{\phi}_{NO}}{\partial \hat{t}} \right\rangle = -\left\langle \hat{U}_i \right\rangle \frac{\partial \left\langle \hat{\phi}_{NO} \right\rangle}{\partial \hat{x}_i} - \frac{\partial}{\partial \hat{x}_i} \left\langle \hat{U}_i'' \hat{\phi}_{NO}'' \right\rangle - Da_{NO} \left(\left\langle \hat{\phi}_{NO} \right\rangle \left\langle \hat{\phi}_{O_3} \right\rangle + \left\langle \hat{\phi}_{NO}'' \hat{\phi}_{O_3}'' \right\rangle \right)$$

Angle brackets '< >' represent the ensemble-averaged value.

NO Transport in Reactive Plume – Budget Analysis

- Assume the pseudo-steady state condition

$$-\langle \hat{U}_i \rangle \frac{\partial \langle \hat{\phi}_{NO} \rangle}{\partial \hat{x}_i} - \frac{\partial}{\partial \hat{x}_i} \langle \hat{U}_i'' \hat{\phi}_{NO}'' \rangle - Da_{NO} \left(\langle \hat{\phi}_{NO} \rangle \langle \hat{\phi}_{O_3} \rangle + \langle \hat{\phi}_{NO}'' \hat{\phi}_{O_3}'' \rangle \right) = 0$$

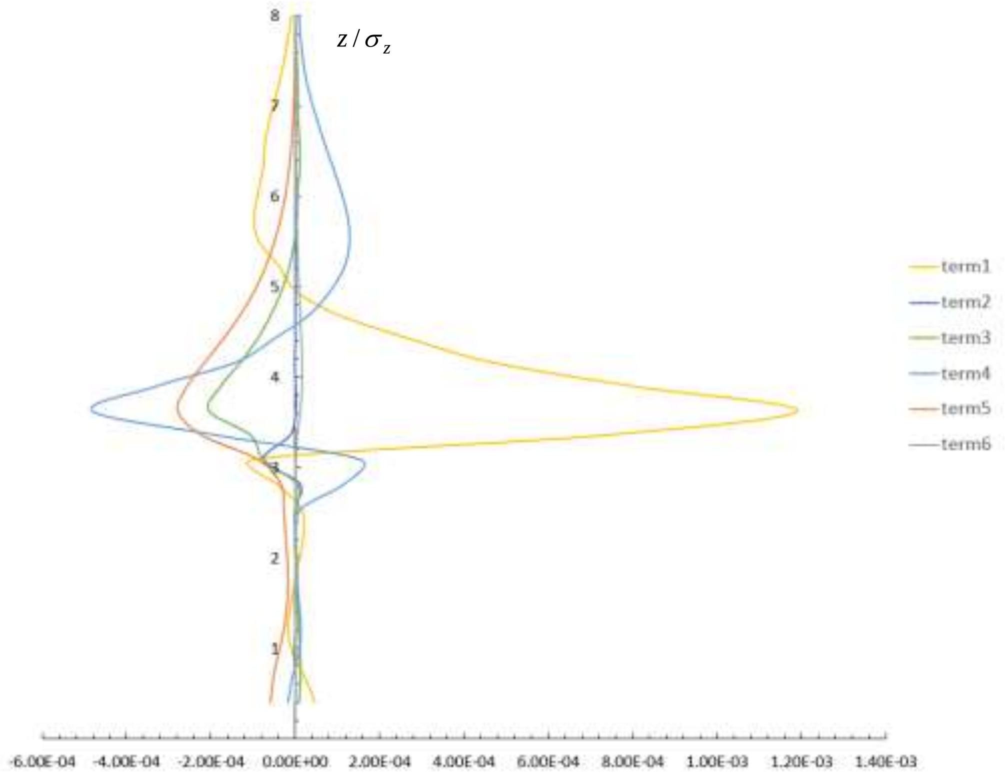
- For 2D flow over street canyon

$$-\langle \hat{u} \rangle \frac{\partial \langle \hat{\phi}_{NO} \rangle}{\partial \hat{x}} - \langle \hat{w} \rangle \frac{\partial \langle \hat{\phi}_{NO} \rangle}{\partial \hat{z}} - \frac{\partial}{\partial \hat{x}} \langle \hat{u}'' \hat{\phi}_{NO}'' \rangle - \frac{\partial}{\partial \hat{z}} \langle \hat{w}'' \hat{\phi}_{NO}'' \rangle - Da_{NO} \langle \hat{\phi}_{NO} \rangle \langle \hat{\phi}_{O_3} \rangle - Da_{NO} \langle \hat{\phi}_{NO}'' \hat{\phi}_{O_3}'' \rangle = 0$$

Term 1 | Term 2 | Term 3 | Term 4 | Term 5 | Term 6

- The first two terms are the advection terms(term1 and term2), two in the middle are the turbulent diffusion terms(term3 and term 4), the last two terms are the chemistry terms(term 5 and term 6).

Budget Analysis in AR=1,$[O_3]_0$=100 ppb in different x location



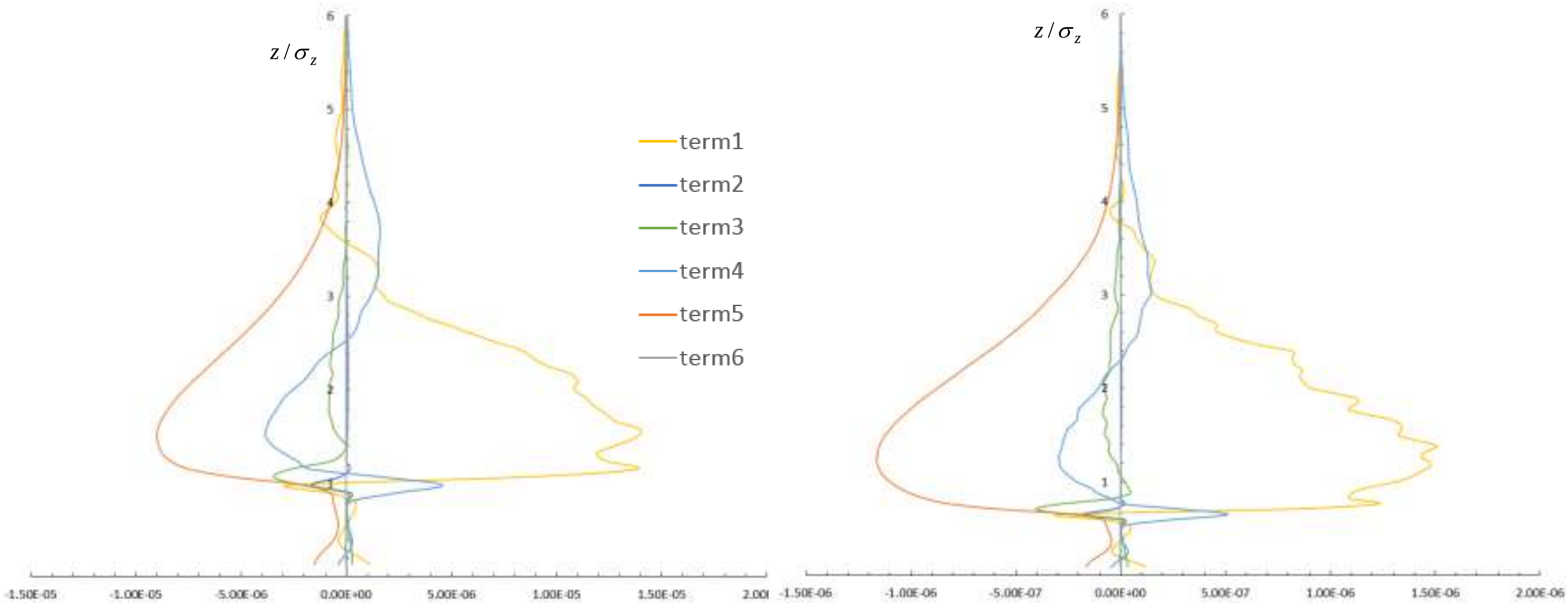
x=5.5

- In the near field, close to the pollutant source, the advection in x direction is important for the pollutant transport.
- Term2 is the advection in Z direction, which is neglectable except at the roof level.
- Term6 is the fluctuation part of the chemistry term, which is also neglectable comparted to others.
- Below the roof level, the chemistry term shift between positive value and negative value, implying the complicated mixing, circulation and chemistry inside the street canyon.

$$-\langle\hat{u}\rangle\frac{\partial\langle\hat{\phi}_{NO}\rangle}{\partial\hat{x}}-\langle\hat{w}\rangle\frac{\partial\langle\hat{\phi}_{NO}\rangle}{\partial\hat{z}}-\frac{\partial}{\partial\hat{x}}\langle\hat{u}''\hat{\phi}''_{NO}\rangle-\frac{\partial}{\partial\hat{z}}\langle\hat{w}''\hat{\phi}''_{NO}\rangle-Da_{NO}\langle\hat{\phi}_{NO}\rangle\langle\hat{\phi}_{O_3}\rangle-Da_{NO}\langle\hat{\phi}''_{NO}\hat{\phi}''_{O_3}\rangle=0$$

Term 1 | Term 2 | Term 3 | Term 4 | Term 5 | Term 6

Budget Analysis in AR=1,$[O_3]_0$=100 ppb in different x location

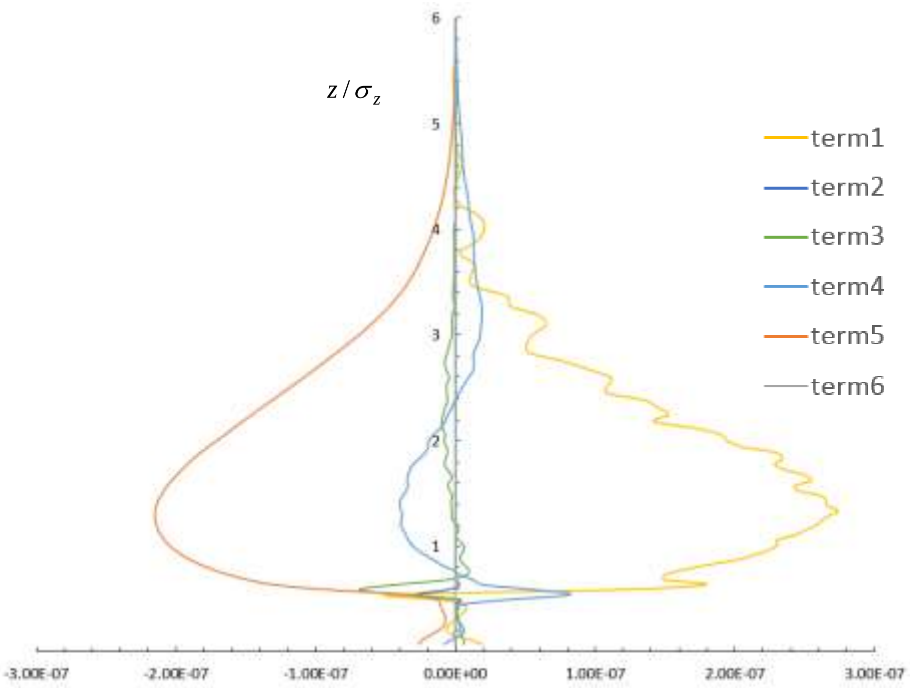


x=25.5 x=45.5

$$-\langle\hat{u}\rangle\frac{\partial\langle\hat{\phi}_{NO}\rangle}{\partial\hat{x}}-\langle\hat{w}\rangle\frac{\partial\langle\hat{\phi}_{NO}\rangle}{\partial\hat{z}}-\frac{\partial}{\partial\hat{x}}\langle\hat{u}''\hat{\phi}''_{NO}\rangle-\frac{\partial}{\partial\hat{z}}\langle\hat{w}''\hat{\phi}''_{NO}\rangle-Da_{NO}\langle\hat{\phi}_{NO}\rangle\langle\hat{\phi}_{O_3}\rangle-Da_{NO}\langle\hat{\phi}''_{NO}\hat{\phi}''_{O_3}\rangle=0$$

Term 1 | Term 2 | Term 3 | Term 4 | Term 5 | Term 6

Budget Analysis in AR=1,$[O_3]_0$=100 ppb in different x location



x=65.5

- In the far field, chemistry term becomes more and more important compared with the advection.
- The diffusion term (both in z and x direction) fluctuates along the vertical height.
- The advection in Z direction(term 2) and term6 is still neglectable comparted to other part.
- The diffusion , advection and chemistry term correlated with each other. Chemistry can not be ignored in the pollutant removal.

$$-\langle\hat{u}\rangle\frac{\partial\langle\hat{\phi}_{NO}\rangle}{\partial\hat{x}}-\langle\hat{w}\rangle\frac{\partial\langle\hat{\phi}_{NO}\rangle}{\partial\hat{z}}-\frac{\partial}{\partial\hat{x}}\langle\hat{u}''\hat{\phi}''_{NO}\rangle-\frac{\partial}{\partial\hat{z}}\langle\hat{w}''\hat{\phi}''_{NO}\rangle-Da_{NO}\langle\hat{\phi}_{NO}\rangle\langle\hat{\phi}_{O_3}\rangle-Da_{NO}\langle\hat{\phi}''_{NO}\hat{\phi}''_{O_3}\rangle=0$$

Term 1 | Term 2 | Term 3 | Term 4 | Term 5 | Term 6

Conclusion

- A NO-O_3 chemistry model is developed to investigate the mechanism of chemically reactive pollutant transport.
- LES data are validated by the wind tunnel experiments.
- Using the modified Gaussian plume model, chemically reactive pollutant still fits in the region above the plume center in the far field.
- Budget analysis shows the correlation between the advection term, diffusion term and chemistry term, implying the significance of chemistry in the pollutant removal.

Thank you !

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