The notion of vanishing-moment recovery (VMR) functions was introduced in [1] for the construction of compactly supported tight frames with two generators having the maximum order of vanishing moments as determined by the given refinable function, such as the $m^{th}$ order cardinal $B$-spline $N_m$. The notion of sibling frames is also introduced to achieve additional properties such as symmetry (or anti-symmetry) and “shift-invariance”, while the generators of the dual frame can be constructed using the same refinable function. For $N_m$, it turns out that symmetry can be achieved for even $m$, and anti-symmetry for odd $m$. Again for $N_m$, minimum support and “shift-invariance” can be attained by considering the two frame generators with two-scale symbols $2^{-m}(1 - z)^m$ and $2^{-m}z(1 - z)^m$, and properly defined generators of the dual frame. Related results were independently obtained in [2].

The construction of compactly supported tight frames is based on two new methods: first, the design of a VMR function that gives a positive definite matrix with Laurent polynomial entries is performed based on known facts about the related transfer operator. Secondly, a new factorization technique for positive definite Laurent polynomial matrices is introduced. This second technique, among other results, is discussed in the talk by W. He.

Keywords: tight frame, wavelet frame, vanishing moments

References

[1]. Chui, C. K., W. He, and J. Stöckler, Compactly supported tight and sibling frames with maximum vanishing moments, preprint.