Properties of Locally Linearly Independent Refinable Function Vectors

Gerlind Plonka
Department of Mathematics
University of Duisburg, Germany
E-mail: plonka@math.uni-duisburg.de

A univariate compactly supported function vector \( \Phi = (\phi_1, \ldots, \phi_r)^T \) is said to be locally linearly independent if for any open set \( A \subseteq \mathbb{R} \)
\[
\sum_{\nu=1}^{r} \sum_{j \in \mathbb{Z}} c_{\nu,j} \in \phi_\nu(-j) \equiv 0 \quad \text{on} \quad A \quad \text{implies} \quad c_{\nu,j} = 0 \quad \forall j \in K_{\phi_\nu}(A),
\]
where \( j \) is in \( K_{\phi_\nu}(A) \) if \( \phi_\nu(-j) \neq 0 \) on \( A \).

The function vector \( \Phi \) is said to be globally linearly independent if in the above definition \( A = \mathbb{R} \).

Hence, local linear independence of \( \Phi \) implies global linear independence. For refinable functions \( (r = 1) \) with dilation parameter 2 local and global linear independence are equivalent. However, for \( r > 1 \) this is not longer true.

The description of local linear independence is rather complicated. However, in a recent paper, Goodman, Jia and Zhou found necessary and sufficient conditions for local linear independence of function vectors which only use the mask coefficients of the refinable vector. In this approach, it is shown that it suffices to show local linear independence on dyadic subintervals of \([0, 1]\).

We want to derive some interesting properties of locally linearly independent function vectors. We also consider function vectors being locally linearly independent over special intervals and study connections with locally and globally linearly independent vectors. A couple of examples will illustrate the theory.