Estimates for the Crest Factor Based on Oversampled Trigonometric Polynomials

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In modern digital communication, a channel input signal is generally synthesized as a linear combination of basis functions whose coefficients are bearing the information that is to be transmitted. In the popular orthogonal frequency division multiplexing (OFDM) communications system, the signal $s$ is expanded in terms of an orthogonal trigonometric basis with restricted base band, i.e., up to a modulation factor we may assume that $s$ is a trigonometric polynomial of degree at most $n$ ($s \in T_n$ for short).

One of the problems of OFDM based transmission is a large peak-to-average ratio (PAR) or, equivalently, a large crest factor (CF) for signals given by

$$CF := \sqrt{\text{PAR}} := \frac{\|s\|_{\infty}}{\|s\|_2} := \frac{\max_{t \in [0,2\pi]} |s(t)|}{\left(\frac{1}{2\pi} \int_0^{2\pi} |s(t)|^2 \right)^{1/2}}.$$

Estimating this CF essentially results in the following equivalent problem: Given the set of $N$ equidistant sampling points in $[0, 2\pi]$, $\Theta_N := \left\{ t_k = k \frac{2\pi}{N} \mid k = 0, \ldots, N-1 \right\}$,

determine the optimal constants $c_{n,N} > 0$ in the inequality

$$\|s\|_{N,\infty} \leq \|s\|_{\infty} \leq c_{n,N} \|s\|_{N,\infty} \quad \text{for all } s \in T_n$$

where $\|s\|_{N,\infty} := \max_{t \in \Theta_N} |s(t)|$.

In this paper we seek estimates for $c_{n,N}$ with methods from classical Approximation Theory. For example, we obtain

$$c_{n,N} \leq \sqrt{\frac{N}{N - 2n}} \quad \text{for } N \geq 2n + 1.$$

We also develop a characterization of $(n, N)$-extremal polynomials (for which the right-hand side estimate in (*) is sharp) in terms of alternation properties, and we present a few cases where the constants $c_{n,N}$ can be determined explicitly.

This is joint work with Götz Pfander and Georg Zimmermann.