A Theory of Gabor Multipliers

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Gabor theory may be understood as the branch of time-frequency analysis which is based on the use of Weyl-Heisenberg families, obtained from a given Gabor atom \( g \) by means of TF-shifts along some lattice \( \Lambda \), \( g_\lambda = \pi(\lambda)g \). Let us assume that \( g \) is a tight Gabor atom with respect to some given TF-lattice \( \Lambda \), i.e. that any \( f \in L^2(\mathbb{R}^d) \) has an \( L^2 \) convergent representation

\[
f = \sum_{\lambda \in \Lambda} \langle f, g_\lambda \rangle g_\lambda,
\]

with \( l^2 \)-coefficients. An operator is called Gabor multiplier if it is of the form

\[
T_m f = \sum_{\lambda \in \Lambda} m(\lambda) \langle f, g_\lambda \rangle g_\lambda,
\]

and \( (m(\lambda))_{\lambda \in \Lambda} \) is called its upper symbol. The theory of Gabor multipliers is concerned with the questions such as:

- What are the properties of \( T_m \), given \( g, \Lambda \) and \( m \)? When does one obtain a Hilbert Schmidt operator (for example)?

- On which Banach spaces (besides \( L^2 \)) are Gabor multipliers bounded, or establish isomorphism between different such spaces?

- Which operators can be well approximated by Gabor multipliers (and how)? What about an (approximate) symbolic calculus?

- What are the properties of the mapping from the upper symbol \( m \) to the operator \( T_m \), for fixed \( (g, \Lambda) \) and what is dependence on \( g \) resp. \( \Lambda \)?