The sampling theory says that if a function $f(x)$ satisfies certain conditions, then it can be reconstructed by its sampled values at some discrete points. For physical reasons, e.g., the inertia of the measurement apparatus, it is difficult to measure the value of a signal precisely at time $x$. In practice, only a local average near $x$ can be measured. Specifically, let $\{x_k\}$ be an increasing real sequence such that $x_k \to \pm \infty$ as $k \to \pm \infty$. Then the sampled values will be $\langle f, u_k \rangle$ for some collection of averaging functions $\{u_k\}$ satisfying that

$$\text{supp } u_k \subset [x_k - \frac{\delta}{2}, x_k + \frac{\delta}{2}], \quad u_k \geq 0, \quad \text{and } \int u_k(x)dx = 1.$$  

It is clear that from local averages one should obtain at least a good approximation of the original signal if $\delta$ is small enough. Wiley, Butzer and Lei studied the approximation error when local averages are used as sampled values.

Furthermore, Gröchenig proved that if $x_{k+1} - x_k \leq \delta < \frac{1}{\sqrt{2}}$, then every band-limited function $f \in B := \{f : f \in L^2(\mathbb{R}) \text{ and supp } f \subset [-\frac{1}{2}, \frac{1}{2}] \}$ is uniquely determined by local averages $\langle f, u_k \rangle$ around $x_k$ and $f$ can be reconstructed by an iterative algorithm. Gröchenig’s result works for arbitrary averaging functions while the sampling rate is greater than the Nyquist rate. Later, Feichtinger and Gröchenig proved that if $\delta := \sup_{k \in \mathbb{Z}}(x_{k+1} - x_k) < \frac{1}{2}$, then every $f \in B$ is uniquely determined by

$$f(x) = \frac{1}{y_k - y_{k-1}} \int_{y_{k-1}}^{y_k} f(x)dx,$$

where $y_k = \frac{x_k + x_{k+1}}{2}.$

We improve Gröchenig’s result and give the optimal upper bounds for the support length of averaging functions with respect to both regular and irregular sampling points. For shift invariant subspaces generated by cardinal B-splines, we also give the optimal upper bound for the support length of averaging functions with irregular sampling points, which generalizes Aldroubi-Gröchenig’s irregular sampling theorem.