Hilbert Pairs of Wavelet Frames

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Recently, research on two distinct types of wavelet frames have emerged, each with its own characteristics. The dual-tree discrete wavelet transform (DWT) [1] is based on the concatenation of two wavelet bases, where the two wavelets are designed to form a Hilbert transform pair, \( \psi_g(t) = \mathcal{H}\{\psi_h(t)\} \) [4]. A second type of wavelet frame is based on a single scaling function and two distinct wavelets. In [2, 3] we considered the design of such systems that are analogous to Daubechies’ wavelets — that is, the design of wavelets of minimal support satisfying certain polynomial properties. The wavelets can be designed so the integer translates of one wavelet fall midway between the integer translates of the other, \( \psi_1(t) \approx \psi_2(t - 0.5) \), hence the term double-density DWT.

While these two types of wavelet frames are similar in some respects (they are both nearly shift-invariant and they are both redundant by a factor of 2 in 1D), they also have basic differences. In this talk we present (approximate) Hilbert transform pairs of wavelet frames that have the advantages of both the double-density DWT and the dual-tree DWT. The DWT based on these wavelet frames can then be called the double-density dual-tree DWT. There are four wavelets, \( \psi_{h,i}(t), \psi_{g,i}(t), i = 1, 2 \), where \( \psi_{g,i}(t) \approx \mathcal{H}\{\psi_{h,i}(t)\} \). Moreover, the two wavelets \( \psi_{h,0}(t) \) and \( \psi_{h,1}(t) \) are off-set from one another by one half and \( \psi_{g,1}(t) \) likewise, \( \psi_{h,1}(t) \approx \psi_{h,2}(t - 0.5) \); \( \psi_{g,1}(t) \approx \psi_{g,2}(t - 0.5) \). The design procedure is based on spectral factorization, paraunitary extension, and the flat-delay filter. The solutions have zero moments and compact support.

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References


