Irregular Sampling Problems and Fast Algorithms Associated with Motion Analysis

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This work addresses a group-theoretic framework that provides a unified way to deal with both irregular sampling problems and fast computational requirements associated with motion transformations embedded in image sequences. Any observable motion transformation can be modeled by means of Lie algebras. Lie algebras define kinematics and provide all the infinitesimal generators to describe the geometry and the kinematical parameters according to the properties of the space under analysis. The construction proceeds with the computation of Lie groups and their representations in the function spaces where motion is to be analyzed. Moving patterns in image sequences undergo a sampling which is function of the relative position of the object and the sensor array. To solve this problem, it is effective to consider motion as a smooth invertible time-warping transformation. Important applications are related to this topic. Let us mention the focalization on selected moving areas characterized by a specific scale and a specific kinematic. Focalization and selective reconstruction can be performed either for analysis purpose with interpolation, prediction, and de-noising or for coding purpose with transmission of limited areas of interest. The Shannon sampling theorem and its generalizations as Kramer and Parzen theorems apply in this context with Clark’s theorem. Clark’s theorem shows that signals formed by warping band-limited signals admit formulae for reconstruction from samples. Furthermore, the warping operators that lift a given pattern up to a trajectory are chosen as unitary irreducible and square-integrable group representations. These operators bring important tools to motion-selective analysis and reconstruction, namely continuous wavelets, frames, discrete wavelet transforms, motion-compensated convolutions and reproducing kernel subspaces. Two examples will be treated with motion at constant translational velocity and angular velocity. As an important result, this group-theoretic framework provides motion-specific fast parallelizable Cooley-Tukey FFT-like structures and motion-specific fast frame-based algorithms. These tools are needed to perform motion estimation, motion-selective tracking and reconstruction of noisy non-uniformly sampled moving patterns.

Introduction

The technique of developing warping models relies on the physical structure of motion which involves Lie algebras or Lie groups. The warping operators are in fact constructed as a Lie group representations i.e. operators in the Hilbert space $H = L^2(\mathbb{R}^2 \times \mathbb{R}, d\tilde{k} d\omega)$ of the signals. \tilde{k} and \omega stand respectively for
the spatial and temporal frequency. Warping involves the deformation of the affine group into a group of motion [5,6] along with the associated continuous wavelets, frame, convolutions and discrete wavelets. These warping transformations require to be generated by invertible operators, to compose one with the other and to preserve the band-limitedness of the still signals. How to build such warping operator? The answer stays in the following choice. Lie group Representations are Unitary Irreducible (UIR) operators in $H = L^2(\mathbb{R}^2 \times \mathbb{R}, dkd\omega)$ defined from a group homomorphism i.e. a on-to-one mapping from the group element $g \in G$ to operator $\Pi_g$ in the Hilbert space $H = L^2(\mathbb{R}^2 \times \mathbb{R}, dkd\omega)$: $g \in G \rightarrow \Pi_g$ such that $\Pi_{g_1}\Pi_{g_2} = \Pi_{g_1g_2}$ and $\Pi_{g^{-1}} = T_{g}^{-1}$; then $\Pi_e = I_H$. Moreover, when this warping operator is square-integrable, it provides a strong structure for signal analysis, decomposition and reconstruction. This structure is made of continuous wavelet transform and frames operator along with reproducing kernel spaces, convolutions, discrete wavelet transforms or orthonormal bases, and in a weaker sense, Riesz bases. Square-integrable warping operators preserve the signal band-limitedness. Two examples of motion warping are considered. The first concerns translational velocity $\vec{v}$ (called Galilean transformation) and the second introduces the angular velocity $\mu$. For the time-warpings defined here above, the analysis and reconstruction structures directly derived from motion-based groups are equivalent to warping the corresponding structures defined from the usual affine multidimensional group into space-time transformations.

The characters of the group representations i.e. the specific Fourier kernel derived from the computation of our group theoretic framework for each specific kinematic define properties that are similar to those known for the usual Fourier transform [3]. For each kinematic, we can find similar properties that provide the Poisson summation formula. Using the Chinese remainder theorem, motion-specific fast parallelizable Cooley-Tukey FFT-like algorithms can be derived.

The group-theoretic construction provides the a priori adequate frames that make the motion-selective reconstruction problem well-posed [1,2] since the construction addresses the actual physical properties of the warping process generating the signal. The concept of frames is one tool to tackle efficient reconstruction algorithms. The design motion-compensated filters/convolutions [4] as wavelets, bi-orthonormal bases and Riesz bases is also solved by this theoretical framework.

References


