Projectable Multivariate Wavelets: Separable vs Nonseparable

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Tensor product (separable) multivariate (bi)orthogonal wavelets have been widely used in many applications. On the other hand, non-tensor product (non-separable) wavelets have been extensively argued in the literature to have many advantages over separable wavelets, for example, more freedom in design of non-separable wavelets (such design is typically much more complicated and difficult than the almost plainless design of separable ones) and better frequency analysis. In this talk, we shall shed some new light to the argument between separable and nonseparable wavelets. We introduce a concept of projectable wavelets. Roughly speaking, a projectable wavelet can be replaced by a separable wavelet without loss of desirable properties such as spatial localization, smoothness and vanishing moments. It turns out that many nonseparable multivariate wavelets proposed in the literature are projectable wavelets. As a consequence of projectable wavelets, we shall prove that in any dimension, there is no continuous axis-symmetric real-valued compactly supported dyadic orthogonal wavelet and there is no continuous symmetric dyadic coiflets. A coset by coset (CBC) algorithm is a simple method to obtain multivariate biorthogonal wavelets. Finally, we shall demonstrate that CBC algorithm achieves certain optimal properties for projectable multivariate wavelets. Related papers can be downloaded at the above web site.