Orthogonal Polynomials on the Unit Ball of $\mathbb{R}^n$

Yuan Xu
Department of Mathematics
University of Oregon
Eugene, OR 97403-1222
email: yuan@math.uoregon.edu

Although the study of orthogonal polynomials of several variables can be traced back to the work of Hermite, the subject lags far behind that of orthogonal polynomials of one variable. Even for classical orthogonal polynomials on the regular regions (for example, cube, ball, simplex, sphere, and the entire $\mathbb{R}^d$), new properties are still being discovered and several questions are resolved only recently.

In this talk, we use orthogonal polynomials on the unit ball as an example to explain questions and results in the subject, and discuss some recent development. Starting with the weight function $W(x) = (1 - \|x\|^2)^{\alpha - 1/2}$, we present several orthogonal bases, reproducing kernels, differential equations, all given in explicit formulae. We will discuss questions about orthogonal expansion, norm of orthogonal projection operator, and summability. We will also discuss various extensions (to more general weight functions and to closely related weight functions on simplex and sphere) and open questions.