Nonnegative Linearization of Orthogonal Polynomials and Group Representations

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The harmonic analysis on the unit circle essentially depends on the multiplicative property of the characters, or the product formula for the cosines

\[ 2 \cos m\theta \cos n\theta = \cos(m - n)\theta + \cos(m + n)\theta. \]

Studying harmonic analysis of compact matrix groups \( SO(n) \) (the unit circle \( SO(1) \) is the only commutative instance among them) leads to another class of special functions called Gegenbauer or ultraspherical polynomials. These functions show up naturally as matrix coefficients of irreducible unitary representations. We will exhibit that these functions satisfy a certain nonnegative linearization formula, a counterpart of that for the cosines.

It turns out that other orthogonal polynomial systems, not necessarily associated with groups, also can admit nonnegative linearization. We will give general condition when it may occur. Also we will show that basing on the nonnegative linearization one can build a convolution structure, similar to that of \( \ell^1(\mathbb{N}) \), and even obtain some pointwise estimates for the polynomials.