On the Global Existence of Solution of Prandtl’s System

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In this paper we establish a global existence of weak solutions to the 2-
dimensional Prandtl’s system for unsteady boundary layers in the class consid-
ered by O. A. Oleinik provided that the pressure is favorable is satisfied. The
main results of Oleinik and her co-workers can be summarized as that there
exists a unique (for short-time if $L$ is given and fixed, and for arbitrary time
if $L$ is small) classical smooth solution to the initial-boundary value problem
provided that the initial data satisfy conditions

$$U(x,t) > 0, \quad u_0(x,t) > 0, \quad u_1(y,t) > 0;$$

and

$$\partial_y u_0(x,y) > 0, \quad \partial_y u_1(x,y) > 0.$$  

One of the open problems listed at the end of book by Oleinik and Samokhin is:
what are the conditions ensuring the global in times existence and uniqueness
of solution for arbitrary given $L$?

The main purpose of this paper is to establish the global (in time) existence
of weak solution to the problem for arbitrary finite $L$ and data satisfying

$$U(x,t) > 0, \quad u_0(x,t) > 0,$$

$$u_1(y,t) > 0, \quad u_0(x,t) \leq 0;$$

and

$$\partial_y u_0(x,y) > 0, \quad \partial_y u_1(x,y) > 0.$$  

and provided that the pressure is favorable, that is,

$$\partial_y p(t,x) \leq 0.$$  

This generalizes the local well-posedness results due to Oleinik. For the proof,
we introduce a viscous splitting method so that the asymptotic behavior near the
fluid can be estimated more accurately by methods applicable to the degenerate
parabolic equations. This is a jointed work with Z.P. Xin.