An Asymptotically Periodic Schrödinger Equation with Indefinite Linear Part

Gongbao Li\(^2\) and Andrzej Szulkin\(^1\)

\(^2\)Wuhan Institute of Physics and Mathematics, The Chinese Academy of Sciences, P. R. China
\(^1\)Department of Mathematics, Stockholm University, Sweden
E-mail:  \(^2\)ligb@wipm.whcnc.ac.cn

We consider the Schrödinger equation \(-\Delta u + V(x)u = f(x,u)\), where \(V\) is periodic and \(f\) asymptotically periodic in the \(x\)-variables, 0 is in a spectral gap of \(-\Delta + V\) and \(f\) is either asymptotically linear or superlinear as \(|u| \to \infty\). We show that this equation has a solution \(u \in H^1(\mathbb{R}^N), u \neq 0\).

The first author was supported in part by NSFC and Academy of Finland. The second author was supported in part by the Swedish Natural Science Research Council.