On the Stationary Solution of the Mathematical Model for Grain Boundary Grooving

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In this talk, we will present some stationary solution for nonlinear partial differential equation called Mullins Equation which occurs in the theory of grain boundary grooving:

\[
u_t = -C^E_1(u)(1 + u^2)^{1/2}\exp(-C^F_2(u)\frac{u_{xx}}{(1 + u^2)^{3/2}}) + C^C_1(u)(1 + u^2)^{1/2}.
\]

The main tool, which we can use, is the admissibility property between weighted continuous function spaces for the integral operator as follows:

\[
T_t x(t) = -\int_t^\infty e^{\xi(t-s)} F(x(s), y(s)) ds,
\]

\[
T_t y(t) = \xi e^{\xi t} + \int_0^t e^{\xi(t-s)} F(x(s), y(s)) ds.
\]

From this admissibility we can prove the existence theorem for the special simultaneous differential equation. This existence theorem can be applied for the second order differential equation,

\[
u'' = f(u, u') = \frac{kT(u)(1 + u^2)^{3/2}}{\nu \gamma}\ln\left(\frac{P_0(u)}{P_c}\right).
\]

The solution of this equation is one of the stationary solution for Mullins Equation.

References