A Modified Nonlinear Galerkin Method for Solving Ginzburg-Landau Equation

YUJIANG WU
Department of Mathematics
Lanzhou University, P. R. China
E-mail: myjaw@lzu.edu.cn

For solving the Ginzburg-Landau equation \( u_t - \nu u_{xx} + \kappa u^3 - \gamma u = f \), \( u(0, x) = u_0(x) \), we introduce a kind of modified nonlinear Galerkin method as follows:

\[
\frac{dy_m}{dt} + \nu A y_m + P_m \{ \kappa (y_m^3 + 3 y_m z_m^2) - \gamma y_m \} = P_m f,
\]

\[

\nu A z_m + z_m^{(2)} + (P_{2m} - P_m) \{ \kappa (y_m + z_m^{(1)})^3 - \gamma (y_m + z_m^{(1)}) \} = (P_{2m} - P_m) f,
\]

\[

\nu A z_m^{(2)} + (P_{2m} - P_m) \{ 3 \kappa (y_m + z_m^{(1)})^2 - \gamma (P_m f - A y_m - P_m (\kappa y_m^3 - \gamma y_m)) \} = 0
\]

\[

\nu A z_m^{(1)} + (P_{2m} - P_m) (\kappa y_m^3 - \gamma y_m) = (P_{2m} - P_m) f,
\]

\[
y_m(0) = P_m u_0
\]

The convergence of the modified method is analyzed and some numerical examples are shown in the article.

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