The Hermit-quadratic and Hermit-cubic Finite-element Approximations to the Non-linear Problems

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For the inviscid Burger’s equation written in the flux form the difference schemes in Hermitian finite-element spaces \( V^2 \) and \( V^3 \) are constructed. Several examples of solutions are included. Let us consider the equation: \( \frac{\partial \phi}{\partial t} + \frac{1}{2} \frac{\partial^2 \phi}{\partial x^2} = 0 \). Applying the finite-difference method, it is usually approximated with the differential-difference equation: \( \frac{d\phi}{dt} + \frac{1}{2} \left( \frac{\phi^{s+1}_{i-1} - \phi^{s-1}_i}{2\Delta t} \right) = 0 \). Using the finite-element method in Lagrangian space with chapeau expansion functions \( \phi_i(x) \) one can obtain more complicated non-linear difference scheme: \( \frac{d\phi}{dt} \left( \frac{\phi^{s+1}_{i-1} + \phi^{s+1}_{i+1}}{6} \right) + \frac{1}{\Delta x^2} \left[ - (\phi^s_i + 2\phi^s_{i-1}) \phi^{s+1}_{i-1} - (\phi^s_{i-1} - \phi^s_{i+1}) \phi^{s+1}_i + (\phi^s_i + 2\phi^s_{i+1}) \phi^{s+1}_{i+1} \right] = 0 \), where \( s \)— the number of iteration on the upper time level. The problem will increase, if we use the Hermite-quadratic or Hermite-cubic finite-element approximations

\[ \phi(x,t) = \sum \left[ \phi^f_i(t) \psi_i^f(x) \right] \left[ \psi_i^q(x), \psi_i^c(x) \right] - \text{quadratic or cubic functions} \]

The finite-element schemes for the term \( \frac{1}{2} \frac{\partial^2 \phi}{\partial x^2} \) have very complicated forms. They are solved iteratively. They will be discussed during the presentation.

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