Solutions with Internal Jump for an Autonomous Elliptic System of Bistable Type

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We consider the following system of semilinear elliptic equations

\[
\begin{align*}
-\varepsilon^2 \Delta u & = f(u) - v \quad \text{in } \Omega; \\
\gamma v - \Delta v & = \delta u \quad \text{in } \Omega; \\
u & = v = 0 \quad \text{on } \partial \Omega.
\end{align*}
\]

We assume \( \Omega \) to be a smooth bounded domain in \( \mathbb{R}^N \), with \( N \geq 1 \), while \( \gamma \) is larger than the first eigenvalue of \( -\Delta \) on \( \Omega \) subjected to homogeneous Dirichlet boundary conditions. We take \( \varepsilon > 0 \) and \( \delta \geq 0 \) as parameters. The nonlinearity we assume for simplicity to be \( f(u) = u(u - 1)(a - u) \) with \( 0 < a < 1/2 \), although other more general nonlinearities can also be treated. We observe that the system is coupled in a noncooperative way, and hence is not order preserving. This leads to a richer solution structure. In particular for small \( \delta \geq 0 \) the solutions to (1) are similar to the solutions to the scalar equation

\[
\begin{align*}
-\varepsilon^2 \Delta u & = f(u) \quad \text{in } \Omega; \\
u & = 0 \quad \text{on } \partial \Omega,
\end{align*}
\]

for which it is known that, under certain assumptions on \( \Omega \) and for \( \varepsilon \) small, there exist only two nontrivial solutions. We shall present results which show how this simple solution structure becomes more complex as \( \delta \) increases.