The Limit from the Schrödinger-Poisson to the Vlasov-Poisson Equations with General Data in One Dimension

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Abstract

We will talk about the classical limit of the Schrödinger-Poisson system to the Vlasov-Poisson equations as the Planck constant ϵ goes to zero. This limit is also frequently called "semiclassical limit". The coupled Schrödinger-Poisson system for the wave functions $\{\psi_j^\epsilon(t,x)\}$ are transformed to the Wigner-Poisson equations for a "phase space function" $f^\epsilon(t,x,\xi)$. For the case of the so called "completely mixed state", i.e. $j = 1, 2, \cdots, \infty$, under additional assumptions on the potential, this classical limit has been solved in 1993 by Lions and Paul (Sur les measures de Wigner, *Rev. Mat. Iberoamericana*, **9**(1993)) and, independently, by Markowich and Mauser (*Math. Meth. Mod. Appl. Sci.*, **3**(1993)) with strong assumptions on the initial data.

The so called "pure state" case where only one or a finite number of wave functions $\{\psi_j^\epsilon(t,x)\}$ are considered, has been open up to now.

We prove here for general initial data (pure state as well as mixed state case) of the wave functions in one space dimension that the Wigner measure $f(t,x,\xi)$, which is a weak limit of $f^\epsilon(t,x,\xi)$ as ϵ tends to 0, satisfies the classical 1-d Vlasov-Poisson equations. As a crucial prerequisite, we have improved the decay assumption on the initial data of 1-d Vlasov-Poisson equations in work by Yuxi Zheng and A. Majda, *Comm. Pure Appl. Math.*, **47**(1994) for the existence of global weak solutions with measures as initial data.

The equations we regard are widely used in quantum/classical transport and semiconductor theory as a nonlinear one particle ("mean field") approximation of the linear N electron Schrödinger/Hamilton equation.