

Fundamental Solutions and Nonlinear Stability of Shock Profiles

Yanni Zeng

Department of Mathematicas
University of Alabama at Birmingham
Birmingham, AL 35294
USA

There exist different tools to study nonlinear stability of shock profiles associated with viscous conservation laws or hyperbolic systems with relaxation. These tools include energy method and its varieties, spectral analysis, and direct construction of fundamental solution. For a generic perturbation, however, these tools so far haven't been able to apply to systems of conservation laws with physical viscosity, or hyperbolic systems with relaxation when the equilibrium systems have at least two quasi-linear equations. The difficulty with those systems is intrinsic: Either the whole system or part of it is hyperbolic, and the fundamental solution contains Dirac δ -functions. Here we provide a new approach to direct construction of fundamental solution. The approach is systematic, and can be used to study nonlinear stability of shock profiles for general hyperbolic-parabolic systems of conservation laws and general hyperbolic systems with relaxation. Special examples include compressible Navier-Stokes equations, full MHD equations, Broadwell Model, and viscoelasticity with fading memory.