Existence and nonexistence of global solutions for a class of quasilinear evolution equations with nonlinear damping and source terms

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Abstract

We consider the global existence of solutions and the blowup of solutions to the initial boundary value problem for a class of quasilinear evolution equations with nonlinear damping and source terms

$$u_{tt} - \Delta u_t - \sum_{i=1}^{n} \frac{\partial}{\partial x_i} s(u_{x_i}) + f(u_t) = g(u), \qquad x \in \Omega, t > 0, \qquad (1.1)$$

$$u\big|_{\partial\Omega} = 0, \qquad t > 0, \qquad (1.2)$$

$$u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x), \qquad x \in \Omega, \qquad (1.3)$$

where Ω is a bounded domain in R^N. Equations of type (1.1) are a class of essential nonlinear evolution equations appearing in the models of nonlinear viscoelasticity. Under the assumptions on the growth orders of $s_i (i = 1, \cdots, n), f$ and g, roughly speaking, there growth orders are respectively m+1, $a + 1$ and p+1, where m, a and p are nonnegative real numbers, we prove that problem (1.1)—(1.3) admits a weak global solution as long as either $m \ge p, 2 \le a + 2 \le N(m+2)/(N-m-2)$ and the initial data $u_0 \in W_0^{1,m+2}, u_1 \in L_2$ or $m < p, u_0 \in W$ (potential well), $u_1 \in L_2$ and the initial energy A(0) (>0) is properly small. And when $p > \max\{m, a\}$, under certain conditions on initial data, the solution of problem (1.1)—(1.3) blows up in finite time.