Boundary Conditions for Multi-Dimensional Relaxation Systems

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We consider the initial-boundary value problem (IBVP) for a class of linear relaxation systems in a half space with arbitrary space dimensions. The prototype systems include the relaxation models proposed by Jin and Xin, and Katsoulakis and Tzavaras, and Natalini, which may or may not admit an entropy estimate. Our goal is to understand the boundary layer behavior of the solution of the relaxation IBVP and to establish its asymptotic convergence to the corresponding equilibrium system of hyperbolic conservation laws in the limit of small relaxation rate. The key is to determine the appropriate structural stability conditions, particularly, the formulation of boundary conditions such that the relaxation IBVP is stiffly well-posed or uniformly well-posed independent of the relaxation parameter. Our main contribution is the derivation, in an explicit and easily checkable form, of a stiff version of the classical Uniform Kreiss Condition (and hence referred to as Stiff Kreiss Condition). The Stiff Kreiss Condition is shown to be necessary and sufficient for the stiff well-posedness and the asymptotic convergence of the relaxation IBVP. It is in general more restrictive than the Uniform Kreiss Condition. However for Jin-Xin model, they turn out be equivalent in any space dimension except $n = 1$.