Steady States of Strongly Coupled Prey–Predator Models

Mingxin Wang
Department of Applied Mathematics, Southeast University,
Nanjing 210018, P. R. China, e-mail: mxwang@seu.edu.cn
(This is a cooperation work with X. F. Chen and W. M. Ni)

Let u and v be the densities of prey and predator respectively. Then we have the following prey–predator model:

$$\begin{cases} u_t = \mathrm{div}\,[K_{11}(u,\,v)\nabla u + K_{12}(u,\,v)\nabla v] + u[b_{11}(u-1)(\theta - u) - b_{12}(v-1)], \\ v_t = \mathrm{div}\,[-K_{21}(u,\,v)\nabla u + K_{22}(u,\,v)\nabla v] + v[b_{21}(u-1) - b_{22}(v-1)],\ x \in - \ ,\ t > 0, \\ \frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = 0,\ x \in \partial - \ ,\quad t > 0, \\ u(x,\,0) \geq 0,\ v(x,\,0) \geq 0,\ x \in - \ , \end{cases}$$

where, constants b_{ij} $(i,\,j = 1,\,2)$ are all positive and $\theta > 1$. - is a bounded smooth domain in $\mathbf{R}^N$, and ν is the outward unit normal vector on $\partial -$. In $\mathbf{R}^2_+$, $K_{ij}(u,\,v)$ are differentiable functions. In this model, $J_{u,x} := -K_{11}(u,\,v)\nabla u - K_{12}(u,\,v)\nabla v$ and $J_{v,x} := K_{21}(u,\,v)\nabla u - K_{22}(u,\,v)\nabla v$ are the diffusive fluxes of u and v in the x-direction respectively. The terms $K_{11}(u,\,v)$ and $K_{22}(u,\,v)$ represent the "self–diffusions", it is required that $K_{11}(u,\,v)$ and $K_{22}(u,\,v)$ are positive functions of $u \geq 0$, $v \geq 0$. The terms $K_{12}(u,\,v)$ and $K_{21}(u,\,v)$ are the "cross–diffusions". The condition $K_{12}(u,\,v) \geq 0$ implies that the flux of u in the x–direction is directed toward decreasing population density of v, i.e. the prey avoids the predator, while $K_{21}(u,\,v) \geq 0$ implies that the flux of v in the x–direction is directed toward increasing population density of u, i.e. the predator chases the prey. In this paper we will discuss the existence of positive non-constant steady states caused by the cross–diffusions.