The Interaction of Dirac Particles with Non-Abelian Gauge Fields and Gravitation

Joel Smoller, Lamberto Cesari Chair

Professor of Mathematics, University of Michigan*

Abstract

We study the differential equations which describe the nonlinear coupling of gravity (as described by Einstein's Theory of General Relativity), to both The Dirac Equation, (describing quantum-mechanical particles), and to a Yang/Mills Gauge Field, (describing nuclear forces), the EDYM equations. The highly nonlinear nature of the resulting equations introduces mathematical problems which are very interesting, and are also quite different from those problems arising in the non-relativistic case. First, we show that for arbitrarily weak gravitational interaction, we can find (linearly) stable bound-state (soliton-like) solutions. This is quite surprising since physicists usually ignore gravitational effects in elementary particle theory becaause gravity is an extremely weak force. Our result shows that the inclusion of arbitrarily weak gravitational interaction has a "regularizing" effect on the equations. We also prove that the EDYM equations DO NOT admit (normalized) black-hole solutions. One interpretation of this is that quantum effects deny the existence of stationary, spherically symmetric black-holes. All of the notions mentioned above will be explained during the lecture. This is joint work with Shing-Tung Yau (Harvard), and Felix Finster (Max Plank Institute, Leipzig, Germany).

*Email address: smoller@math.lsa.umich.edu