Asymptotic Behavior of the 1-D Compressible Flow through Porous Media

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In this talk we consider the Cauchy problem for the one-dimensional compressible flow through porous media :

$$(1)\qquad \begin{cases} v_t - u_x = 0, \quad (x,t) \in \mathbf{R} \times \mathbf{R}_+ \\ u_t + p(v)_x = -\alpha u \quad (\alpha > 0 \ : \text{constant}) \\ (v,u)|_{t=0} = (v_0,u_0)(x) \to (v_\pm, u_\pm) \quad \text{as} \quad x \to +\infty. \end{cases}$$

Here, v is the specific volume, so that $v_0(x)$ and $v_\pm$ are assumed to be positive; u is the velocity; $p = p(v) = v^{-\gamma}(\gamma \geq 1)$ is a pressure. The solution (v,u) to (1) time-asymptotically behaves as the diffusion wave $(\tilde{v}, \tilde{u})$ satisfying

$$(2)\qquad \tilde{v}_t - \tilde{u}_x = 0, \quad p(\tilde{v})_x = -\alpha \tilde{u},$$

due to the Darcy's law, which was first investigated by [Hsiao and Liu, Comm. Math. Phys. **143**(1992)], by introducing suitable auxiliary functions. The convergence rate and asymptotic profile were obtained in [Nishihara, J. Diff. Eqs **131**(1996), **137**(1997)] by investigating both the second order hyperbolic equation with linear damping and the corresponding parabolic equation. The relation of those equations is crucial for looking for the precise large-time behavior. So, we also show the L^p -L^q estimate of the difference $V - \phi$

$$(3)\qquad \|(V-\phi)(x,t) - \frac{1}{2}e^{-t/2}(V_0(x+t) + V_0(x-t))\|_{L^p} \leq Ct^{-1-\frac{1}{2}(\frac{1}{q}-\frac{1}{p})}\|V_0, V_1\|_{L^q}$$

for $p \geq q \geq 1$, where V and ϕ are, respectively, the solutions to those euqations of constant coefficients

$$(4)\qquad V_{tt} - V_{xx} + V_t = 0, \quad (V, V_t)|_{t=0} = (V_0, V_1)(x) \in L^q$$

and

$$(5)\qquad -\phi_{xx} + \phi_t = 0, \quad \phi|_{t=0} = (V_0 + V_1)(x) \in L^q.$$

Note that

$$(6)\qquad \|\partial_x^\alpha \partial_t^\beta \phi(x,t)\|_{L^p} \leq Ct^{-\frac{1}{2}(\frac{1}{q}-\frac{1}{p})-\frac{\alpha}{2}-\beta}\|V_0 + V_1\|_{L^q}.$$