

# Stability of traveling waves for the Broadwell model in the half space

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The discrete Boltzmann equation appears in the discrete kinetic theory of rarefied gases. This system of equations describes the motion of gas particles with a finite number of velocities. Thus, it is interesting and important to analyze the behavior of the solution under boundary effect not only as a purely mathematical problem, but also from the physical point of view. In the papers [1, 2, 3], the existence and the stability of a stationary wave are proved for a general system of the discrete Boltzmann equations in the half space  $\{x > 0\}$ .

Following these results, we study the stability of a traveling wave solution with a positive speed for a concrete model of Boltzmann equations, the Broadwell model system:

$$\partial_t F_1 + v \partial_x F_1 = F_2^2 - F_1 F_3, \quad (1a)$$

$$4\partial_t F_2 = -2(F_2^2 - F_1 F_3), \quad (1b)$$

$$\partial_t F_3 - v \partial_x F_3 = F_2^2 - F_1 F_3. \quad (1c)$$

Here,  $v$  is a positive constant and the unknown function  $F_1$ ,  $F_2$  and  $F_3$  represent the mass densities for gas particles moving with the speed  $v$ ,  $0$  and  $-v$  in the  $x$ -direction, respectively. We abbreviate as  $F := (F_1, F_2, F_3)$ . The traveling wave to the system (1) is a solution in the form:  $\tilde{F}(\xi)$  where  $\xi := x - st$  and  $s$  is a positive constant called a traveling wave speed. The spatial asymptotic states should be Maxwellian states. Precisely,  $\tilde{F}(\xi) \rightarrow M^\pm = (M_1^\pm, M_2^\pm, M_3^\pm)$  as  $\xi \rightarrow \pm\infty$ , where  $(M_2^\pm)^2 = M_1^\pm M_3^\pm$ ,  $M_i^\pm > 0$  for  $i = 1, 2, 3$ . Then, we assume the initial data  $F(x, 0)$  converges to the Maxwellian  $M^+$  as  $x \rightarrow \infty$ . Moreover, as the characteristic speeds of (1a) is positive, we prescribe the pure diffusive boundary condition,  $F_1(0, t) = M_1^-$ .

It is shown that  $(F_1, F_2, F_3)(x, t)$  converges to  $(\tilde{F}_1, \tilde{F}_2, \tilde{F}_3)(x - st + \delta)$  uniformly in  $x > 0$  as  $t \rightarrow \infty$ , where  $\delta$  is a certain constant. The asymptotic location parameter  $\delta$  is determined by the boundary data as well as the initial data. This theorem is proved by the standard energy method with the aid of the estimates along the boundary, which is obtained from the diffusive boundary condition.

## References

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