# Existence, Decay and Blow up of Weak Solutions to Some Degenerate Type Davey-Stewartson Equations

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We consider the following degenerate Davey–Stewartson equations as a model of completely integrable systems that generalize the 1D nonlinear Schrödinger equation to higher dimensions

$$i\psi_t + \psi_{xx} = \chi\psi, \tag{1}$$

$$i\phi_t + \phi_{xx} = \chi\phi, \tag{2}$$

$$\chi_y = (|\psi|^2 + \alpha|\phi|^2)_x \tag{3}$$

$$\psi(0,x,y) = \psi_0(x,y), \quad \phi(0,x,y) = \phi_0(x,y), \quad (x,y) \in \mathbf{R}^2, \tag{4}$$

$$\lim_{|x|\to\infty} \psi(t,x,y) = 0, \quad \lim_{|x|\to\infty} \phi(t,x,y) = 0, \quad \lim_{y\to-\infty} \chi(t,x,y) = 0, \tag{5}$$

where $\alpha > 0, \alpha \neq 1, \psi(t,x,y), \phi(t,x,y)$ are complex unknown functions, and $\chi(t,x,y)$ is a real unknown function. We obtain

**Theorem** *For any* $\psi_0, \phi_0 \in L^2(\mathbf{R}^2)$ *satisfying* $\psi_{0x}, \phi_{0x} \in L^2(\mathbf{R}^2)$ *and* $\|\psi_0\|_{L^2}^2 + \alpha\|\phi_0\|_{L^2}^2 < \frac{1}{2}$, (1)–(5) *have a global weak solution.*

*For* $\psi_0, \phi_0 \in \Sigma = \{u \in L^2(\mathbf{R}^2) \,|\, u_x, xu \in L^2(\mathbf{R}^2)\}$ *with* $\|\psi_0\|_{L^2}^2 + \alpha\|\phi_0\|_{L^2}^2 < \frac{1}{2}$, *then* (1)–(5) *have a global weak solution satisfying*

$$\|\psi\|_{L^2(\mathbf{R}_y;L^p(\mathbf{R}_x))} + \|\phi\|_{L^2(\mathbf{R}_y;L^p(\mathbf{R}_x))} \leq C_p t^{-(1/2-1/p)}, \qquad 2 < p \leq \infty.$$

*Assume that* $\psi_0, \phi_0 \in \Sigma$, $(\psi, \phi, \chi)$ *is a solution of* (1)–(5) *satisfying* $\psi, \psi_x, x\psi, \phi, \phi_x, x\phi \in L^2(\mathbf{R}^2)$, *and* $\chi$ *vanishes at infinity. If either one of the following three conditions holds: i.e. either*

(i) $E(0) < 0$, *or*

(ii) $E(0) = 0$, $F(0) > 0$, *or*

(iii) $E(0) > 0$, $F(0) \geq 4\sqrt{E(0)I(0)}$,

*where*

$$\begin{aligned} E(t) &= \int \left(|\psi_x|^2 + \alpha|\phi_x|^2 + \frac{1}{2}\chi(|\psi|^2 + \alpha|\phi|^2)\right) dxdy \equiv E(0), \\ F(t) &= -4\mathrm{Im}\int x(\psi\bar{\psi}_x + \alpha\phi\bar{\phi}_x)\, dxdy, \qquad I(t) = \int x^2(|\psi|^2 + \alpha|\phi|^2)\, dxdy, \end{aligned}$$

*then there exists a* $T^* > 0$ *such that the solution blows up at* $T^*$. *That is,*

$$\liminf_{t\to T^*}(\|\psi_x\|_2 + \alpha\|\phi_x\|_2) = \infty.$$