Global Well-Posedness and Relaxation Limits of a model for Radiating Gas

Corrado Lattanzio

and Pierangelo Marcati
Università degli Studi di L'Aquila,
I-67100 L'Aquila, Italy *

Abstract

We are interested in the study of the following hyperbolic-elliptic model

$$\begin{cases} u_t + \frac{1}{2}(u^2)_x = -q_x \\ -q_{xx} + q = -u_x \end{cases} \tag{1}$$

with $u_0 \in L^1(\mathbb{R}) \cap L^\infty(\mathbb{R})$ as initial condition. This system is known as the simplest mathematical model in the study of radiating gases.

After rewriting the system (1) as the following nonhomogeneous Burgers' equations

$$u_t + \frac{1}{2}(u^2)_x = -u + K * u,$$

where the convolution kernel K is given by $\frac{1}{2}e^{-|x|}$, it is possible to prove the comparison and the L^1 contraction properties for his solutions. Hence, taking advantage of these results, we first show the well-posedness in $L^1 \cap L^\infty$ for the weak entropy solutions of (1). Then, using appropriate transformations, we prove the strong L^1 convergence of the rescaled systems toward their relaxation limits, namely the inviscid and the viscous Burgers' equation.

*Email address: corrado@univaq.it