

Energy dissipation in body-forced turbulence

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Abstract

The bulk rate of energy dissipation in body-force driven steady state turbulence is studied via weak solutions of the incompressible Navier-Stokes equations. We consider flows in volume L^3 in 3 dimensions in the absence of boundaries and derive an *a priori* estimate for the time averaged energy dissipation rate per unit mass, ϵ , without any assumptions on the turbulent fluctuations. We prove

$$\epsilon \leq c_1 \nu \frac{U^2}{\ell^2} + c_2 \frac{U^3}{\ell}$$

where ν is the kinematic viscosity, U is the root mean square (space and time averaged) velocity, and $\ell \leq L$ is the longest length scale in the applied forcing function. The prefactors c_1 and c_2 depend only on the functional shape of the applied body-force and not on its magnitude or any length scales in the body force, the domain or the flow. We also derive both upper *and* lower bounds on ϵ in terms of the Grashof number, a nondimensional measure of the magnitude of the driving force. These estimates are interpreted in terms of the conventional scaling theory of turbulence and discussed in the context of direct numerical simulations and wind tunnel measurements. This is joint work with C. Foias

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