

Exact WKB analysis and Jacobi polynomials with varying nonstandard parameters

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We consider the (monic) Jacobi polynomials $\hat{P}_n^{(\alpha,\beta)}(x)$ with the parameters of the form

$$\alpha = nA, \quad \beta = nB \quad ,$$

where A and B are real parameters satisfying $-1 < A < 0$, $-1 < B < 0$ and $-2 < A + B < -1$. The asymptotic expansion with respect to the degree n for $\hat{P}_n^{(nA,nB)}(x)$ was investigated by A.B.J. Kuijlaars and A. Martínez-Finkelshtein in 2004. In this talk, we will show that we can prove their main theorem by using the relation between the hypergeometric function and WKB solutions which will be given in Tanda's talk. And we will also show that exact WKB analysis allows us to find $\hat{P}_n^{(nA,nB)}(x)$ have the same asymptotic behavior in the case where both A and B are complex parameters satisfying $-1 < \operatorname{Re} A < 0$, $-1 < \operatorname{Re} B < 0$ and $-2 < \operatorname{Re} (A + B) < -1$.

This is a joint work with Mika TANDA and Takashi AOKI.