On Wright's generalized Bessel kernel

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In this talk, we consider the Wright's generalized Bessel kernel $K^{(\alpha,\theta)}(x,y)$ defined by

$$\theta x^{\alpha}\int_0^1 J_{\frac{\alpha+1}{\theta},\frac{1}{\theta}}(ux)J_{\alpha+1,\theta}((uy)^{\theta})u^{\alpha}du, \qquad \alpha>-1, \qquad \theta>0,$$

where

$$J_{a,b}(x)=\sum_{j=0}^{\infty}\frac{(-x)^j}{j!\Gamma(a+bj)}, \qquad a\in\mathbb{C}, \qquad b>-1,$$

is Wright's generalization of the Bessel function. This non-symmetric kernel, which generalizes the classical Bessel kernel (corresponding to $\theta=1$) in random matrix theory, is the hard edge scaling limit of the correlation kernel for certain Muttalib-Borodin ensembles. We show that, if θ is rational, i.e., $\theta=\frac{m}{n}$ with $m,n\in\mathbb{N}$, $gcd(m,n)=1$, and $\alpha>m-1-\frac{m}{n}$, the Wright's generalized Bessel kernel is integrable in the sense of Its-Izergin-Korepin-Slavnov. We then come to the Fredholm determinant of this kernel over the union of several scaled intervals, which can also be interpreted as the gap probability (the probability of finding no particles) on these intervals. The integrable structure allows us to obtain a system of coupled partial differential equations associated with the corresponding Fredholm determinant as well as a Hamiltonian interpretation. As a consequence, we are able to represent the gap probability over a single interval $(0,s)$ in terms of a solution of a system of nonlinear ordinary differential equations.