## Circulants and root location of derivatives of polynomials


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With use of the so-called one-rank perturbation method introduced by Yu. Barkovsky we obtain a curious properties of circulant matrices and improve a theorem by W. Cheung and T. Ng (former de Bruin and Sharma's conjecture) to the following one:

*Let $\lambda_1$, $\lambda_2$, $\ldots$, $\lambda_n$ be the roots of a polynomial $p$ of degree $n \geqslant 2$, and let $w_1$, $w_2$, $\ldots$, $w_{n-1}$ be the roots of its derivative $p'$. If $\sum_{j=1}^{n} \lambda_j = 0$, then*

$$\sum_{k=1}^{n-1} |w_k|^4 \leqslant \frac{n-4}{n} \sum_{j=1}^{n} |\lambda_j|^4 + \frac{1}{n^2} \left( \sum_{j=1}^{n} |\lambda_j|^2 \right)^2 + \frac{1}{n^2} \left| \sum_{j=1}^{n} \lambda_j^2 \right|^2 ,$$

*where equality holds if, and only if, all $\lambda_j$ lie on a straight line passing through the origin of the complex plane.*

We also extend this fact by giving an estimate to $\sum_{k=1}^{n-1} |w_k|^4$ for arbitrary complex polynomials. Some applications to orthogonal polynomials of this fact will be discussed in the talk.