Lagrange expansion theorem along with arbitrary Weighted Dyck path

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This talk is devoted to the following generalization of the classical Lagrange–Bürmann inversion theorem

Theorem $\forall F(x) \in \mathbb{C}[[x]]$ and $\{\phi_n(x) | \forall n \geq 0, \phi_n(0) \neq 0\}$, there always holds

$$F(x) = \sum_{n \geq 0} a_n \frac{x^n}{\prod_{i=0}^{n} \phi_i(x)},$$

where

$$a_n = \sum_{k=0}^{n} [x^k] F(x) \sum_{\mathcal{P} \in \mathcal{P}_{n,n}^{+}(k)} \mathrm{W}\big(\mathcal{P} \big| \phi_0 \to \phi_n\big),$$

where we define the set of Dyck paths by

$$\mathcal{P}_{n,n}^{+}(k) = \{\mathcal{P} = (k \leq j_1 \leq j_2 \leq \cdots \leq j_n \leq n) | j_i \geq i\}$$

and the weight (w.r.t. $\{\phi_n(x)\}_{n \geq 0}$) of any Dyck path

$$\mathcal{P} = (k \leq j_1 \leq j_2 \leq \cdots \leq j_n \leq n)$$

to be the product

$$[x^{j_1 - k}]\phi_0(x) \times [x^{j_2 - j_1}]\phi_1(x) \times \cdots \times [x^{j_n - j_{n-1}}]\phi_{n-1}(x)[x^{n - j_n}]\phi_n(x)$$

denoted in short by

$$W(\mathcal{P} | \phi_0 \to \phi_n).$$

Our result may serve as a common generalization of many expansion theorems such as Carlitz's, Andrews', Garsia–Haiman's and Haglund's q-analogues of the Lagrange–Bürmann inversion theorem, etc.