

# Generalized Freud polynomials

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We use the moments of the generalized Freud weight

$$w_\lambda(x; t) = |x|^{2\lambda+1} \exp(-x^4 + tx^2), \quad x \in \mathbb{R}, \quad \lambda > 0, \quad t \in \mathbb{R}$$

together with a connection between orthogonal polynomials and Painlevé equations to obtain an expression for the recurrence coefficient of monic semiclassical orthogonal polynomials with respect to the even weight  $w_\lambda(x; t)$ . We derive other properties such as the second-order linear ordinary differential equation and differential-difference equation satisfied by generalized Freud polynomials. We analyze the asymptotic behavior of generalized Freud polynomials when the parameter  $t$  involved in the semiclassical perturbation of the weight tends to  $\pm\infty$  and obtain asymptotic results for the polynomials and recurrence coefficient as the degree  $n$  tends to infinity. We also investigate some properties of the zeros of generalized Freud polynomials.