Gibbs Chinese restaurants, Abel-Riemann-Liouville operators and Beta identities derived from stable subordinators

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It is known that random partitions of the integers $[n] := \{1, \ldots, n\}$, can be generated by a process of discovery of excursion intervals of Brownian motion or more general Bessel processes. That is to say, processes whose inverse local time follows a stable subordinator of index $\alpha \in (0, 1)$. Remarkably, the distribution of these partitions, and its most tractable extensions, may be expressed in terms of a two parameter Chinese restaurant process (CRP). This CRP may also be generated by exchangeably sampling n variables from what is now known as a Pitman-Yor process (named in [2]), which has applications in Bayesian statistics and machine learning and arises in numerous areas of probability. Jim Pitman (and subsequently with Gnedin), showed that further conditioning on the local time or inverse local time up till time one, leads to a general class of random partitions that have infinite Gibbs (product) structure as a characterizing feature. In particular, the Brownian case $\alpha = 1/2$, may be expressed in terms of Hermite or confluent hypergeometric functions. (For an overview see Pitman [3, in particular section 4]). In 2007, in response to a question raised by Pitman [3, 4.3.3, p. 87], we ([1]) showed for the general α case that one can express the relevant quantities in terms of Fox H and Meijer G functions. However recently, we recognized there was much more to the story, and this involves decompositions of certain Abel-Riemann-Liouville operators and other interesting consequences.

References

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