

## Zeros of Ultraspherical and pseudo-Ultraspherical polynomials

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The pseudo-ultraspherical polynomial of degree  $n$  is defined by  $\tilde{C}_n^{(\lambda)}(x) = (-i)^n C_n^{(\lambda)}(ix)$  where  $C_n^{(\lambda)}(x)$  is the ultraspherical polynomial. We discuss the orthogonality of finite sequences of pseudo-ultraspherical polynomials  $\{\tilde{C}_n^{(\lambda)}\}_{n=0}^N$  for different values of  $N$  that depend on  $\lambda$ . We discuss applications of Wendroff's Theorem and use an identity linking the zeros of the pseudo-ultraspherical polynomial  $\tilde{C}_n^{(\lambda)}$  with the zeros of the ultraspherical polynomial  $C_n^{(\lambda')}$  where  $\lambda' = \frac{1}{2} - \lambda - n$  to prove that when  $1 - n < \lambda < 2 - n$ , two (symmetric) zeros of  $\tilde{C}_n^{(\lambda)}$  lie on the imaginary axis.