Overcoming the Curse of Dimensionality for Certain Hamilton-Jacobi (HJ) Equations Arising in control Theory and Elsewhere

Jerome Darbon and Stanley Osher *
Department of Mathematics, The University of California (Los Angeles) USA
Email: sjo@math.ucla.edu

It is well known that certain HJ PDE's play an important role in analyzing continuous dynamic games and control theory problems. The cost of standard algorithms, and, in fact all PDE grid based approximations is exponential in the space dimension and time, with huge memory requirements. Here we propose and test methods for solving a large class of HJ PDE relevant to optimal control without the use of grids or numerical approximations. Rather we use the classical Hopf formulas for solving initial value problems for HJ PDE. We have noticed that if the Hamiltonian is convex and positively homogeneous of degree one that very fast methods (related to those used in compressed sensing) exist to solve the resulting optimization problem. We seem to obrtain methods which are polynomial in dimension. We can evaluate the solution in very high dimensions in between $10^{(-4)}$ and $10^{(-8)}$ seconds per evaluation on a laptop. The method requires very limited memory and is almost perfectly parallelizable.
In addition, as a step often needed in this procedure, we have developed a new and equally fast and efficient method to find, in very high dimensions, the projection of a point exterior to a compact set A onto A. We can also compute the distance to such sets much faster than fast marching or fast sweeping a algorithms.
The term "curse of dimensionality" was coined by Richard Bellman in 1957 when he did his pioneering work on dynamic optimization.