

Special Functions from a Riemann-Hilbert Analysis of the Freud Weight

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We consider a pair of special functions, u_β and v_β , defined respectively as the solutions to the integral equations

$$u(x) = 1 + \int_0^\infty \frac{K(t)u(t)dt}{t+x} \quad \text{and} \quad v(x) = 1 - \int_0^\infty \frac{K(t)v(t)dt}{t+x}, \quad x \in [0, \infty),$$

where $K(t) = \frac{1}{\pi} \exp(-t^\beta \sin \frac{\pi\beta}{2}) \sin(t^\beta \cos \frac{\pi\beta}{2})$ for $\beta \in (0, 1)$. In this note, we establish the existence and uniqueness of u_β and v_β which are bounded and continuous in $[0, +\infty)$. Also, we show that a solution to a model Riemann-Hilbert problem in Kriecherbauer and McLaughlin [*Int. Math. Res. Not.*, 1999] can be constructed explicitly in terms of these functions. A preliminary asymptotic study is carried out on the Stokes phenomena of these functions by making use of their connection formulas.

Several open questions are also proposed for a thorough investigation of the analytic and asymptotic properties of the functions u_β and v_β , and a related new special function G_β .

This is a joint work with R. Wong.