Learning local basis for ray based finite element method for high frequency Helmholtz equation

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We present a ray-based finite element method (Ray-FEM) for the high-frequency Helmholtz equation in smoothly varying media. The method first uses low frequency wave to probe the medium and learn local ray directions based on numerical micro-local analysis on the low frequency wave field. The local ray information is then incorporated into the finite element basis for solving the high frequency Helmholtz equation. Our method requires the minimum degrees of freedom, i.e., a fixed number of grid points per wavelength, to achieve both stability and expected accuracy for the high-frequency Helmholtz solution without the usual pollution effect. Under some assumptions, the resulting matrices can be solved using fast methods in an empirical complexity $O(\omega^d)$, where ω is the frequency and d is the dimension.