Efficient numerical solution of the time-dependent Ginzburg–Landau equations for superconductors with geometric defects

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Numerical investigations and analyses on Galerkin FEMs for the time-dependent Ginzburg–Landau (TDGL) equations have been done extensively in the last several decades. In the vortex dynamics of the TDGL equations, the domain shape plays an important role. Of high interest in the study of superconductivity is the influence of a geometric defect in the superconductor to the vortex generation and motion. Numerical experiments show that the solution of conventional Galerkin finite element methods may not converge to a physical one when the geometry of the superconductor is nonsmooth.

We present a Galerkin-mixed method for the TDGL equations with Lorentz gauge. The proposed method enjoys many advantages over existing methods. We establish optimal error estimates of the proposed method for problems in a more general domain, including a nonconvex polygon. The method provides the same order of optimal accuracy for the density function ψ, the magnetic potential $\mathbf{A}$, the magnetic field $\sigma = \mathbf{curl}\,\mathbf{A}$, the electric potential $\operatorname{div}\mathbf{A}$ and the current $\mathbf{curl}\,\sigma$. Extensive numerical experiments in both two and three dimensional spaces, including complex geometries with defects, are presented to illustrate the accuracy and stability of the scheme. Our numerical results also show that the proposed method provides more realistic predictions for the vortex dynamics of the TDGL equations in nonsmooth domains.