Semi-implicit Methods for Phase Field Equations

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Recent results in the literature provide computational evidence that stabilized semi-implicit time-stepping method can efficiently simulate phase field problems involving fourth-order nonlinear diffusion, with typical examples like the Cahn-Hilliard equation and the thin film type equation. The up-to-date theoretical explanation of the numerical stability relies on the assumption that the derivative of the nonlinear potential function satisfies a Lipschitz type condition, which in a rigorous sense, implies the boundedness of the numerical solution. In this work we remove the Lipschitz assumption on the nonlinearity and prove unconditional energy stability for the stabilized semi-implicit time-stepping methods. It is shown that the size of stabilization term depends on the initial energy and the perturbation parameter but is independent of the time step. The corresponding error analysis is also established under minimal nonlinearity and regularity assumptions.