A unified framework for harmonic analysis of functions on directed graphs and changing data

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Diffusion geometry is a recent powerful tool for analyzing high dimensional, unstructured data by embedding it on a low dimensional manifold. We present a general framework for studying harmonic analysis of functions in the settings of various emerging problems in the theory of diffusion geometry. The starting point of the now classical diffusion geometry approach is the construction of a kernel whose discretization leads to an undirected graph structure on an unstructured data set. We study the question of constructing such kernels for directed graph structures, and argue that our construction is essentially the only way to do so using discretizations of kernels. We then use our previous theory to develop harmonic analysis based on the singular value decomposition of the resulting non–self–adjoint operators associated with the directed graph. Next, we consider the question of how functions defined on one space evolves to another space in the paradigm of changing data sets recently introduced by Coifman and Hirn. While the approach of Coifman and Hirn require that the points on one space should be in a known one–to–one correspondence with the points on the other, our approach allows the identification of only a subset of landmark points. We introduce a new definition of distance between points on two spaces, construct localized kernels based on the two spaces and the landmark points, and study the evolution of smoothness of a function on one space to its lifting to the other space via the landmarks. We develop novel mathematical tools that enable us to study these seemingly different problems in a unified manner.