Singularities in the Complex Spatial Plane of a Vortex Sheet with Blob Regularization

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We study in this work a periodic version of Krasny's vortex blob equation, whose solutions are known to converge to a classical weak solution of the Euler equation upon vanishing blob size. The equation is analytically continued into the complex spatial plane of its parametrization variable, and for a specific initial condition used by Meiron et al., a simplified far-field equation is derived which is then studied using Taylor expansions in small time and small blob size. Like the unregularized vortex sheet studied by Cowley et al., the solutions of the far-field equation develop singularities at $t = 0^+$. These singularities propagate in from $\mathrm{i}\infty$, and local analysis suggests that they are square-root branch points, possibly infinitely many in quantity. All analytic results are confirmed in a subsequent numerical study, and certain properties of the complex singularities, like their locations and strengths, are investigated numerically in some detail. It appears that all singularities of the blob-regularized vortex sheet collapse to the $\frac{3}{2}$-power singularity of the unregularized vortex sheet as the blob size vanishes, but the precise way in which this collapse happens is yet to be determined.