Eulerian Approaches based on the Level Set Method for Visualizing Continuous Dynamical Systems

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We propose efficient Eulerian numerical approaches based on the Level Set Method for constructing flow maps in continuous dynamical systems. We first apply these methods to compute the finite time Lyapunov exponent (FTLE), the finite size Lyapunov exponent (FSLE) and also a related infinitesimal size Lyapunov exponent (ISLE). There are several advantages of the proposed approach. Unlike the usual Lagrangian-type computations, the resulting method requires the velocity field defined only at discrete locations. No interpolation of the velocity field is needed. Also, the method automatically stops a particle trajectory in the case when the ray hits the boundary of the computational domain. The computational complexity of the algorithm is $O(\Delta x^{d-1})$ with d the dimension of the physical space. Since there are the same number of mesh points in the extended phase space (the $x-t$ space), the computational complexity of the proposed Eulerian approach is optimal in the sense that each grid point is visited for only $O(1)$ time. Based on the techniques developed, we then discuss numerical approach to extract invariant sets in continuous dynamical systems in the extended phase space. We extend the idea of ergodic partition and propose a concept called coherent ergodic partition for visualizing ergodic components in a continuous flow.