# Efficient and Stable Exponential Time Differencing Runge-Kutta Methods for Phase Field Elastic Bending Energy Models

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The Willmore flow formulated by phase field dynamics based on the elastic bending energy model has been widely used to describe the shape transformation of biological lipid vesicles. In this talk, we present some efficient and stable numerical methods for simulating the unconstrained phase field Willmore dynamics and the phase field Willmore dynamics with fixed volume and surface area constraints. The proposed methods can be high-order accurate and are completely explicit in nature, by combining exponential time differencing Runge-Kutta approximations for time integration with spectral discretizations for spatial operators on regular meshes. We also incorporate novel linear operator splitting techniques into the numerical schemes to improve the discrete energy stability. In order to avoid extra numerical instability brought by use of large penalty parameters in solving the constrained phase field Willmore dynamics problem, a modified augmented Lagrange multiplier approach is proposed and adopted. Various numerical experiments are performed to demonstrate accuracy and stability of the proposed methods.