Hankel Determinants for a Singular Complex Weight and the First and Third Painlevé Transcendents

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We consider polynomials orthogonal with respect to a varying perturbed Laguerre weight $e^{-n(z-\log z+t/z)}$ for $t<0$ and z on certain contours in the complex plane. When the parameters n, t and the degree k are fixed, the Hankel determinant for the singular complex weight is shown to be the isomonodromy τ-function of the Painlevé III equation. When the degree $k=n$, n is large and t is close to a critical value, inspired by the study of the Wigner time delay in quantum transport, we show that the double scaling asymptotic behaviors of the recurrence coefficients and the Hankel determinant are described in terms of a Boutroux tronquée solution to the Painlevé I equation. Our approach is based on the Deift-Zhou nonlinear steepest descent method for Riemann-Hilbert problems.
This a joint work with Shuai-Xia Xu and Yu-Qiu Zhao.