

Zeros of exceptional Hermite polynomials

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Exceptional orthogonal polynomials were introduced by Gomez-Ullate, Kamran and Milson as polynomial eigenfunctions of second order differential equations with the remarkable property that some degrees are missing, i.e., there is not a polynomial for every degree. However, they do constitute a complete orthogonal system with respect to a weight function that is typically a rational modification of a classical (Hermite, Laguerre, Jacobi) weight function.

For the case of exceptional Hermite polynomials these weights take the form $W(x)^{-2}e^{-x^2}$ where $W(x)$ is a Wronskian determinant constructed out of a finite number of Hermite polynomials. The cases of interest are when W has no zeros on the real line. It is known that in those cases most of the zeros of the exceptional Hermite polynomials are real. Our new result is that they asymptotically distribute themselves in the same way as the zeros of usual Hermite polynomials, and in addition, that the finitely many non-real zeros tend to the zeros of W as the degree tends to infinity.

This is joint work with Robert Milson (Dalhousie University)