Zeroes of the Swallowtail Integral

David Kaminski
University of Lethbridge, Canada
Email: kaminski@uleth.ca

The swallowtail integral $S(x,y,z) = \int_{-\infty}^{\infty} \exp[i(u^5 + xu^3 + yu^2 + zu)]\,du$ is one of the so-called canonical integrals used in optics, and plays a role in the uniform asymptotics of integrals exhibiting a confluence of up to four saddle points. In a 1984 paper by Connor, Curtis and Farrelly, the authors make a number of remarkable observations regarding the zeroes of $S(x,y,z)$, including that its zeroes occur on lines in xyz-space, and that the zeroes of $S(0,y,z)$ lie along the line $y = 0$. These assertions are based on numerical evidence and the asymptotics of $S(0,0,z)$. This talk examines these assertions more completely and provides additional detail of the structure of the zeroes of $S(x,y,z)$.