Simulations of waves in heterogeneous media

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Numerical modeling of wave propagation in heterogeneous media is important in many applications. Due to the complex nature, direct numerical simulations on the fine grid are prohibitively expensive. It is therefore important to develop efficient and accurate methods that allow the use of coarse grids. In this paper, we present a multiscale finite element method for wave propagation on a coarse grid. The proposed method is based on the Generalized Multiscale Finite Element Method (GMsFEM). To construct multiscale basis functions, we start with two snapshot spaces in each coarse-grid block where one represents the degrees of freedom on the boundary and the other represents the degrees of freedom in the interior. We use local spectral problems to identify important modes in each snapshot space. These local spectral problems are different from each other and their formulations are based on the analysis. To our best knowledge, this is the first time where multiple snapshot spaces and multiple spectral problems are used and necessary for efficient computations. Using the dominant modes from local spectral problems, multiscale basis functions are constructed to represent the solution space locally within each coarse block. These multiscale basis functions are coupled via the symmetric interior penalty discontinuous Galerkin method which provides a block diagonal mass matrix, and, consequently, results in fast computations in an explicit time discretization. Our methods' stability and spectral convergence are rigorously analyzed. Numerical examples are presented to show our methods' performance. We also test oversampling strategies. In particular, we discuss how the modes from different snapshot spaces can affect the proposed methods' accuracy. This is a joint work with Yalchin Efendiev and Wing Tat Leung in Texas A&M University. The research of Eric Chung is partially supported by Hong Kong RGC General Research Fund (Project: 400411).