Analysis and Control of some Models for Fluid-structure Interaction

Enrique Zuazua
Departamento de Matemáticas
Universidad Autónoma, Madrid, Spain
E-mail: enrique.zuazua@uam.es

In this lecture we shall discuss some simplified models for fluid-structure interaction. From a mathematical point of view these models are characterized by the coupling of two different components in which parabolic and hyperbolic equations are coupled, through, possibly, finite dimensional differential equations for solid masses.

First we shall analyse a one-dimensional model for the interaction between a fluid and a solid mass. The fluid is governed by the viscous Burgers equation and the solid mass, which shares the velocity of the fluid, is accelerated by the difference of pressure at both sides of it. We describe the asymptotic behavior of solutions for integrable data using energy estimates and scaling techniques. We prove that the asymptotic profile of the fluid is a self-similar solution of the Burgers equation with an appropriate mass and we describe the parabolic trajectory of the solid mass. We also consider the case of a finite number of masses and we show that they may not collide in finite time.

We then consider a linearized model coupling a wave with a heat equation. We dicuss the main properties of this model in what concerns, well-posedness and regularizing effects. A comparison with the classical system of thermoelasticity will be given empashizing the new aspects related to the fact that the two equations do not hold in the same domain but rather in two domains separated by an interface. The rate of decay and controllability of this system will also be discussed. As we shall see, the nature of the hyperbolic-parabolic coupling makes this issue complex and very much dependent on the geometry of the two domains involved in the model.